Consider a composite natural number n greater or equal to 4. We aim at proving that there is at least one natural number r such as $n-r$ and $n+r$ are both primes. For obvious reasons $r < n-2$. Such a number r will be called a "primality radius" of n.

Now let's define the number $ord_C(n)$, which depends on n, in the following way : $ord_C(n) := \pi\left(\sqrt{2n-3}\right)$, where $\pi(x)$ is the number of primes less or equal to x, and $\sqrt{x}$ is the square root of x. $n+r$ is a prime only if for all prime p less or equal to $\sqrt{2n-3}$, p doesn't divide $n+r$. There are exactly $ord_C(n)$ such primes. The number $ord_C(n)$ will be call the "natural configuration order" of n.

Now let's define the "k-order configuration" of an integer m, denoted $C_k(m)$, as the sequel $(m \bmod 2, m \bmod 3, ..., m \bmod p_k)$. For example $C_4(10) = (10 \bmod 2, 10 \bmod 3, 10 \bmod 5, 10 \bmod 7) = (0, 1, 0, 3)$. We call $C_{ord(n)}(n)$ the "natural configuration" of n.

A sufficient condition to make r a primality radius of n is that for all integer i such that $0 < i < 1 + ord_C(n)$, $(n-r) \bmod p_i$ differs from 0 and $(n+r) \bmod p_i$ differs from 0. If this statement is true, r will be called a "potential typical primality radius" of n. More over, if $r < n-2$, then r will be called a "typical primality radius" of n.

Let's define $N_1(n)$ as the number of potential typical primality radii of n less than $P_{ord_C(n)}$, where $P_{ord_C(n)} = 2 \times 3 \times ... \times p_{ord_C(n)}$. It is easy to give the expression of $N_1(n)$.

Now let's define $N_2(n)$ as the number of typical primality radii of n. One can show that $N_2(n) = n\frac{N_1(n)}{P_{ord_C(n)}} + O(1)$ (Landau's notation).

The boy whose nickname is qg77 in the link I gave earlier (a brilliant high-school pupil) showed that :

$$n\frac{N_1(n)}{P_{ord_C(n)}} > c\frac{n}{\log^2 n}(1+o(1))$$

where c is a positive constant.

So that $N_2(n)$ tends to infinity as n grows, and the Goldbach's conjecture is true for sufficiently large numbers.