INTRODUCTION

For the purpose of my proof I use the consequence of the theory of the Continued Fractions:

For the equation ax + by = c, (a, b) = 0

$x=cq_{n-1}-tb$ (3)

$y=cp_{n-1}-ta$ (4)

where p_{n-1} is numerator and q_{n-1} is denominator of n-1 convergent of a/b expansion as simple continued fraction, with an even number n of partial quotients;
and t=0,$\pm1, \pm2, \pm3$….

Here are some citations from the Olds' book "Continued Fractions" about continued fractions and their use for solution of equations:

Any rational number we may write in the following form:

$$\frac{9}{7} = 1 + \frac{2}{7} = 1 + \frac{1}{\frac{7}{2}} = 1 + \frac{1}{3 + \frac{1}{2}} = 1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1}}} .$$

An expression of the form

$$a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{a_3 + \cfrac{b_3}{a_4 + \ddots}}} \qquad (1.5)$$

Is called continued fraction

Continued fractions of the following form:

$$a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \ddots}}} ,$$

Where the first term a_1 is usually a positive or negative integer (but could be zero), and where the terms $a_2, a_3, a_4, \cdots$ are positive integers, are called simple continued fractions.

Continued fractions of the form:

$$a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}}},$$

With only a finite number of terms

$a_1, a_2, a_3, \cdots, a_n$.

is called finite (or terminating) simple continued fraction.

A rational number is a fraction of the form p/q where p and q are integers with q ≠ 0. Every rational fraction, or rational number, can be expressed as a finite simple continued fraction (is proved in the book)

[the book gives examples how continued fraction expansions can be calculated for different rational fractions]

THEOREM 1.1. *Any finite simple continued fraction represents a rational number. Conversely, any rational number p/q can be represented as a finite simple continued fraction; with the exceptions to be noted below, the representation, or expansion, is unique.*

[is proofed in the book]

$$(1.19) \quad \frac{p}{q} = a_1 + \frac{1}{a_2 +}\,\frac{1}{a_3 +}\cdots\frac{1}{+ a_n} = [a_1, a_2, a_3, \cdots, a_n].$$

[this is much more convenient way to writing continued fractions, used in the book.]

The *uniqueness* of the expansion (1.19) follows from the manner in which the a_i's are calculated. This statement must be accompanied, however, by the remark that once the expansion has been obtained we can always modify the *last* term a_n so that the number of terms in the expansion is either *even* or *odd*, as we choose. To see this, notice that if a_n is greater than 1 we can write

$$\frac{1}{a_n} = \frac{1}{(a_n - 1) + \frac{1}{1}},$$

THEOREM 1.2. *Any rational number p/q can be expressed as a finite simple continued fraction in which the last term can be modified so as to make the number of terms in the expansion either even or odd.*

From the properties of convergents, described in the book follows some results for Diophantine Equations described in this book and used for solution of some linear indeterminate equations with two unknown integers. Namely:

THEOREM 1.3. *The numerators* p_i *and the denominators* q_i *of the* ith *convergent* c_i *of the continued fraction* $[a_1, a_2, \ldots . a_n]$ *satisfy the equations*

$$p_i = a_i p_{i-1} + p_{i-2}, \quad q_i = a_i q_{i-1} + q_{i-2}, \qquad (i = 3, 4, 5, \cdots, n) \tag{1.28}$$

with the initial values

$$p_1 = a_1, \quad p_2 = a_2 a_1 + 1, \quad q_1 = 1, \quad q_2 = a_2. \tag{1.29}$$

And:

THEOREM 1.4. *If* $p_i = a_i p_{i-1} + p_{i-2}$ *and* $q_i = a_i q_{i-1} + q_{i-2}$ *are defined as in Theorem* 1.3, *then*

$$p_i q_{i-1} - p_{i-1} q_i = (-1)^i, \qquad \textit{where } i \geq 0.$$

And here is the citation from the book about equation ax + by = c:

[on the next page]

2.5 The General Solution of $ax + by = c, \quad (a, b) = 1$

The discussion of this equation is similar, except for some minc changes, to that of the equation $ax - by = c$. Still assuming tha a and b are positive integers, we first find a particular solution (the equation

$$ax + by = 1, \qquad (a,b) = 1$$

To do this, expand a/b as a simple continued fraction with an *eve* number of partial quotients. From the table of convergents rea off p_{n-1} and q_{n-1}. Then

$$aq_{n-1} - bp_{n-1} = 1,$$

as before. The trick now is to write the given equation $ax + by =$ in the form

$$ax + by = c \cdot 1 = c(aq_{n-1} - bp_{n-1}).$$

Rearrange terms to obtain

$$a(cq_{n-1} - x) = b(y + cp_{n-1}). \tag{2.24}$$

This shows that b divides the left side of the equation; but $(a, b) = 1$ so b cannot divide a. Therefore b divides $cq_{n-1} - x$, so tha there is an integer t such that

$$cq_{n-1} - x = tb, \tag{2.25}$$

or

$$x = cq_{n-1} - tb. \tag{2.26}$$

Substitute (2.25) into (2.24) to get

$$a(tb) = b(y + cp_{n-1}),$$

and solve for y to obtain

$$y = at - cp_{n-1}. \tag{2.27}$$

Conversely, for any integer t, a direct substitution of (2.26) and (2.27) into $ax + by$ gives

$$\begin{aligned} ax + by &= a(cq_{n-1} - tb) + b(at - cp_{n-1}) \\ &= acq_{n-1} - tab + tab - bcp_{n-1} \\ &= c(aq_{n-1} - bp_{n-1}) = c \cdot 1 = c, \end{aligned}$$

so the equation $ax + by = c$ is satisfied. Thus the general solution of the equation $ax + by = c$ is

$$\begin{aligned} x &= cq_{n-1} - tb \\ y &= at - cp_{n-1} \end{aligned} \qquad t = 0, \pm 1, \pm 2, \pm 3, \cdots . \tag{2.28}$$

These are the equations I use for my proof.

And here is the link to the book: Olds C.D. Continued Fractions (http://www.ms.uky.edu/~sohum/ma330/files/Continued%20Fractions.pdf)

PROOF

Beal's Conjecture: If $A^x + B^y = C^z$, (1) where A, B, C, x, y, and z are positive integers and x, y and z are greater than 2, then A, B and C must have a common prime factor.

Let's suppose such case, for $A^x + B^y = C^z$ is possible, when A, B and C have not common factors. For the purpose of proof we use the consequence of the theory of the Continued fractions:

For the equation ax+by=c,

Where a, b and c have not common factors,

x=cq_{n-1}-tb

y= ta - cp_{n-1}

where pn-1 is numerator and qn-1 is denominator of n-1 convergent of a/b expansion as simple continued fraction, with an even number n of partial quotients;
and t=0,$\pm$1, $\pm$2, $\pm$3….

We may consider equation Ax+By=Cz as AAx-1+ BBy-1 = Cz , where A=x , Ax-1 = a, B = y, By-1 = b and Cz = c; so we have:

$A = C^z q_{n-1} - tB^{y-2}$ (1)

And

$B = tA - C^z p_{n-1}$ (2)

And also considering Ax+By=Cz as $A^2 A^{x-2} + B^2 B^{y-2} = C^z$, where A^2 = x, $A^{x-2} =$ a, $B^2 =$ y, $B^{y-2} =$ b and Cz = c

We have

$A^2 = C^z q^{|}_{n-1} - t^{|} B^{y-2}$ (3)

$B^2 = C^z p^{|}_{n-1} - t^{|} A^{x-2}$ (4)

So, from (1) and (3), we have $A = C^z q_{n-1} - tB^{y-2}$ and $A^2 = C^z q^{|}_{n-1} - t^{|} B^{y-2}$

We can write:

$(C^z q_{n-1} - tB^{y-2})^2$ = $C^z q^{|}_{n-1} - t^{|} B^{y-2}$

Or

$C^{2z}q_{n-1}^2 - 2C^z q_{n-1} t B^{y-1} + t^2 B^{(y-1)2} = C^z q_{n-1}^{|} - t^{|} B^{y-2}$

$C^{2z}q_{n-1}^2 - 2C^z q_{n-1} t B^{y-1} - C^z q_{n-1}^{|} = -t^2 B^{(y-1)2} - t^{|} B^{y-2}$

$2C^z q_{n-1} t B^{y-1} + C^z q_{n-1}^{|} - C^{2z} q_{n-1}^2 = t^2 B^{(y-1)2} + t^{|} B^{y-2}$

$C^z \left(2q_{n-1} t B^{y-1} + q_{n-1}^{|} - C^z q_{n-1}^2\right) = B^{y-2}(t^2 B^2 + t^{|})$

C and B must not have any common prime factor, and so:

$C^z r = t^2 B^2 + t^{|}$;

or $t^{|} = C^z r - t^2 B^2$ (5)

Where r may be any rational number (not definitely positive),

Now, from (2) and (4) we have:

$B = A^{x-1} t - C^z p_{n-1}$ and $B^2 = A^{x-2} t^{|} - C^z p_{n-1}^{|}$

So,

$(A^{x-1} t - C^z p_{n-1})^2 = A^{x-2} t^{|} - C^z p_{n-1}^{|}$

$A^{(x-1)2} t^2 - 2A^{x-1} t C^z p_{n-1} + C^{2z} p_{n-1}^2 = A^{x-2} t^{|} - C^z p_{n-1}^{|}$

$C^z p_{n-1}^{|} - 2A^{x-1} t C^z p_{n-1} + C^{2z} p_{n-1}^2 = A^{x-2} t^{|} - A^{(x-1)2} t^2$

$C^z \left(p_{n-1}^{|} - 2A^{x-1} t p_{n-1} + p_{n-1}^2\right) = A^{x-2}(t^{|} - t^2 A^x)$

C and A must not have any common prime factor, and so:

$C^z s = t^{|} - t^2 A^2$;

Or $t^{|} = t^2 A^2 + C^z s$ (6)

Where s may be any rational number (not definitely positive)

So, from (5) and (6) we have: $C^z r - t^2 B^2 = t^2 A^2 + C^z s$

$C^z r - C^z s = t^2 B^2 + t^2 A^2$

$C^z (r - s) = t^2 (A^2 + B^2)$

$A^2 + B^2$ could not be divided with C^z when z > 2 and also C and t must not have common divisors, and so, our supposition is not true, such case is impossible and Beal's conjecture may be considered as proven.