V4 PROOF OF GRIMM'S CONJECTURE

Grimm's Conjecture states that for every composite integer sequence x, x+1, x+2, x+3, x+4, …, x+n, there is a unique prime in each composite integer in the prime factorization of x, x+1, x+2, x+3, x+4, …, x+n.

The Fundamental Theorem of Arithmetic states that every positive integer greater than one is either prime or has a unique prime factorization. The standard form for this prime factorization is p1*p2*p3*p4*...*pn.

The composite sequence will be preceded by a prime that is one unit less than its starting value and will end immediately before the next prime that is one unit greater than its ending value:

x-1 (prime), x, x+1, x+2, x+3, x+4, …, x+n, x+n+1 (prime)

Also, the first composite integer in the sequence will be even because it is preceded by an odd prime, and an odd integer plus one is an even integer. This means that every other integer in the composite sequence starting with the first composite integer in the sequence is even. Carrying this idea further, one of every three integers in the composite integer sequence is divisible by three, one of every five integers in the composite integer sequence is divisible by five, and one of every n (n prime) composite integers in the composite integer sequence is divisible by n. It follows the pattern if H is divisible by n, then so is H*n*Z+ (except zero).

A non-unique prime factorization of one of the composite integers in the sequence is the only way for Grimm's Conjecture to be wrong. This will require one of the primes in the prime factorization of one of the composite integers in the sequence to increase its power by at least one:

Let $X=(p1)^r*(p2)^s*(p3)^t*(p4)^u*...*(pn)^w$

Then, one way for an integer in the composite sequence to have non-unique prime divisors is to increase the power of at least one of the prime divisors of X by at least one:

Multiply X by p1 to increase r by one:

$$p1*X=p1^{(r+1)}*(p2)^s*(p3)^t*(p4)^u*...*(pn)^w$$

Multiplying X by any powers of r, s, t, u, v, w,… n will be analogous.

So, using the first composite integer in the sequence already proven to be an even integer, we set this even composite integer equal to X and increase the power of the integer 2 by one unit for 2X:

$$X=2^r*(p1)^s*(p2)^t*(p3)^u*(p4)^v*...*(pn)^w$$

Multiplying X by 2 increases 2^r to 2^(r+1):

2X=2^(r+1)*(p1)^s*(p2)^t*(p3)^u*(p4)^v*...*(pn)^w

(r, s, t, u, v, w, and n are members of the positive integers greater than zero)

(p1, p2, p3, p4,...pn are prime numbers)

We have X and 2X, and there is always a prime according to Bertrand’s Postulate (proven by Chebyshev in 1852) between X and 2X. And this prime will be before 2X. There is at least one prime number before 2X, and the prime with the smallest value of the primes after X and before 2X will end the composite sequence and be less than 2X. And, since all odd primes are greater than two, they will also be outside of the composite sequence when their power is increased by one or more:

2X<3X1<5X2<... PrimeX

Let’s say X1 is divisible by 3, X2 is divisible by 5, X3 is divisible by 7, etc.

Then 2X<3X1<5X2<7X3, …<nXn (n largest prime divisor in sequence).

There will always be prime divisors equal to the closest prime to the number of composite integers in the sequence and all prime numbers below that number of composite integers.

In some cases there is a unique factorization before 2X. This can happen when, for example, we have a square of one of the prime factors:

101^2*103=1050703, 101*103^2=1071509, The difference between these two integers is less than 101^2*103, so 101*103^2 is less than twice 101^2*103. However, using the good minimum number of primes up to x formula (x/(log(x)-1) (x>=5,393):

1,050,703/(log(1,050,703)-1)=75,786

1,071,509/(log(1,071,509)-1)=77,198

This means there are at least 1,412 primes between the two composite integers.

Increasing the prime numbers by a substantial amount in the prime factorization will increase the number of primes between the two composite integers. For example:

7,949^2*7,951=502,396,664,551, 7,949*7,951^2=502,523,069,549

502,396,664,551/(log(502,396,664,551)-1)=18,646,887,227

502,523,069,549/(log(502,523,069,549)-1)=18,651,404,704

This means there are at least 4,517,477 prime numbers between the two composite integers.

There will always be an increase in the number of primes between two composite integers as follows: If we have two composite integers X and Y, with Y>X, then the number of primes between them will increase as follows:

Y/(log(Y)-1) is the minimum number of prime numbers up to Y and X/(log(X)-1) is the minimum number of primes up to X. If Y>X, then log(Y) will increase about +2.3 per power of ten, and log(X) is analogous.

Let's examine the fraction that the two composite integers generated previously are apart:

101^2*103=1,050,703, 101*103^2=1,071,509

The fraction they are apart is 103/101=1.0198…

Evidently every fraction between one and two is the same number of decimal places past the decimal point as the number of digits of the integers or the same number of digits minus one. And those are the only integers that can generate two composite integers with the same prime numbers in the factorization.

Using the formula for minimum number of primes up to x formula: x/(log (x)-1) (x>=5,393) again, we can prove the aforementioned fractions are too big to be before the next prime after x using the fact the natural logarithm increases +2.302585… for every power of ten:

Since we use the square of one of the integers to generate the non-unique prime factorization, we have, for example, using 101^2*103, a seven digit integer.
The generated integer (101*103^2) is 103/101=1.0198… times the first integer (101^2*103). However, the minimum number of primes between the two integers is
{1,071,509/(log(1,071,509)-1)}-{(1,071,509-(20,806))-(/(log(1,071,509-(20,806)))-1)}

Evidently using the aforementioned symmetric natural logarithms and the formula for minimum number of primes up to x proves there is always a prime before the fractions that generate non-unique prime factorizations.

QED