Proof of Goldbach conjecture

Nurlan Gasimli

January 22, 2019

Abstract

In this article, I prove Goldbach conjecture which is one of the oldest problems in number theory. In 1742, German mathematician Christian Goldbach wrote a letter to great mathematician Leonhard Euler and proposed this conjecture. The statement is that every even integer greater than 2 can be expressed as sum of two primes. I show my proof in few steps.

Infinitely many prime numbers

Firstly, I want to show Euler's proof which is about product formula.

$$\prod_{p \in P} \sum_{k \geq 0} \frac{1}{p^k} = \sum_{k \geq 0} \frac{1}{2^k} \times \sum_{k \geq 0} \frac{1}{3^k} \times \sum_{k \geq 0} \frac{1}{5^k} \times \sum_{k \geq 0} \frac{1}{7^k} \times \cdots$$

$$= \sum_{k,\ell,m,n,\cdots \geq 0} \frac{1}{2^k 3^\ell 5^m 7^n \ldots} = \sum_n \frac{1}{n}$$

in the result, every product of primes appears exactly once and so by the fundamental theorem of arithmetic the sum is equal to the sum over all integers.

The sum on the right is the harmonic series, which diverges. The product on the left must also diverge. Since each term of the product is finite, the number of terms must be infinite; therefore, there is an infinite number of primes.

Prime numbers' formula

I show that every prime number (after 2 and 3) can be expressed in two ways. Firstly, 6n-1 and next is 6n+1, n is integer which is > 0. If we look any intervals of 6 numbers, for example, 18-23, 330-335, 1100-1105, it continues to infinity, we understand that there are 3 chances of 6 numbers being prime. If we take 18-23, numbers are below shown:

18 19 20 21 22 23

18, 20 and 22 are even, so that they can not be prime. 21 can be divided by 3 and it also can not be prime. There are two possibilities, 19 and 23. 19 is 6n+1 where n is 3 and 23 is 6m -1 where m is 4. Let's look another example and then I show this possibility in terms of formula.

102 103 104 105 106 107

102, 104 and 106 are even and they can not be prime. If we take other numbers, 105 is divided by 3 but 103 and 107 are primes. If we go back 101 is also prime and we go further 109 is also prime. 101 is 6n-1, 103 is 6n+1 and 107 is 6m-1, 109 is 6m+1. If we look that to infinity all primes, every prime can be shown one of 4 formulas which are indicated above. But It is not vital that all primes must be consecutive. 6n+1 may be prime and 6m-1 may not be prime but every prime numbers can be shown one of 4 formulas.

Sum of two primes

If we consider prime numbers formula and sum these formulas we get these equations.

6n+1+6n+1=12n+2

6n+1+6n-1=12n-2

6n+6n=12n

All are even numbers and we get same equations with m.

Additionally,

6n+1+6m+1=6*(m+n)+2

6n-1+6m-1=6*(m+n)-2 and etc. All are even numbers.

Can all even numbers be shown as sum of these primes?

Yes.

If we look interval of 12 numbers, for example 36-47, 240-251 and it continues to infinity.

36 37 38 39 40 41 42 43 44 45 46 47

We only look even numbers and we express any numbers in one of 4 formulas.

36=12n

38=12n+2

40=6*(m+n)-2

42=6*(m+n)

44=6*(m+n)+2

46=12m-2

If we consider infinitely prime numbers, we understand that all even numbers are surrounded by these formulas.

Finally, we get that:

P1+P2=2n

P=prime

N=all integers greater than 1.

So Goldbach Conjecture has been proven.