The Collatz Conjecture – Deconstructed (an example of experimental mathematics)

Introduction.

The Collatz conjecture can be stated very simply : for any positive integer if it is odd then multiply it by 3 and add 1. If it is even then divide by 2. Repeat. The end point of this process so far has inevitably been 1. Why.

I have discovered two equations that allow me to follow any path created by the Collatz conjecture or in fact create any and all of the infinite possible paths (given infinite time to do so) starting from 1.

Most people select numbers and calculate the path back to 1. This is the top down approach. My approach starts at 1 and can create any and all paths that can be formed by the Collatz conjecture. This is the bottom up approach.

Finding a particular number from my equations is akin to finding a tree in the woods. If there is a path marked to the tree you can follow the path and get to the tree. If there is no path then you can find many trees by wandering around. In every step of the calculation using my equations there is a new set of infinite number of possible 'trees'.

The solution is important for 'solving' the Collatz conjecture but as it is probably new to the mathematical establishment it may be more important for opening up a way to look at other 'hailstone' functions.

Here is my approach:

Powers of 2 will always give 1 when divided by 2 a sufficient number of times – so at present I will ignore them.

Even numbers not being powers of 2 will inevitably give rise to an odd number on division by 2 for a

sufficient number of times – so at present I will ignore them.

So I am really only interested in odd numbers.

Mathematically the expression of interest is as follows:

Eqn 1: 3*A+1=N : A is odd, N is even but not a power of 2.

Transposing terms

Eqn 2: A = (N - 1)/3

Now N is actually an odd number times 2 raised to some power : so what we have is

Eqn 3: A = (B*(2^n) - 1)/3 : B is odd

This implies that if we substitute various integers for B and varying values for n then we can calculate values of A.

Using a spreadsheet then a general table for A can be calculated.

Table 1.

	p =	0	1	2	3	4	5
	B =	1	3	5	7	9	11
n	2^n						
1	2	0.33	1.67	3.00	4.33	5.67	7.00
2	4	1.00	3.67	6.33	9.00	11.67	14.33

3	8	2.33	7.67	13.00	18.33	23.67	29.00
4	16	5.00	15.67	26.33	37.00	47.67	58.33
5	32	10.33	31.67	53.00	74.33	95.67	117.00
6	64	21.00	63.67	106.33	149.00	191.67	234.33
7	128	42.33	127.67	213.00	298.33	383.67	469.00
8	256	85.00	255.67	426.33	597.00	767.67	938.33
9	512	170.33	511.67	853.00	1194.33	1535.67	1877.00

However there are a large number of non integer values in the table. Discarding columns that only have non integer values you arrive at a much reduced table that still has non integer values scattered through it.

Visual inspection of this table suggests that there are two functions controlling the calculation of integer values. One function involves odd powers of n and a seperate function involves even powers of n.

The two expressions for A are:

Eqn 4: A1 = ((6p + 1)*(2^2n)-1)/3 n contained in the set (1,2,3,.......) p contained in the set (0,1,2,3,.......)

Eqn 5: A2 = ((6p - 1)*2^(2n+1)-1)/3 n contained in the set (0,1,2,3,.......) p contained in the set (1,2,3,.......)

Note the differences in ranges for p and n in the two expressions.

Using various values for n and p generates two sets of odd numbers [A1] and [A2].

Table 2. Computed values for [A2] using equation 5.

	p =	1	2	3	4	5	6
	B1 =	5	11	17	23	29	35
n	2^(2n+1)						
0	2	3	7	11	15	19	23
1	8	13	29	45	61	77	93
2	32	53	117	181	245	309	373
3	128	213	469	725	981	1237	1493
4	512	853	1877	2901	3925	4949	5973

Table 3. Computed values for [A1] using equation 4.

	p =	0	1	2	3	4	5	6
	B2 =	1	7	13	19	25	31	37
n	2^(2*n)							

1	**4**	1	9	17	25	33	41	49
2	**16**	5	37	69	101	133	165	197
3	**64**	21	149	277	405	533	661	789
4	**256**	85	597	1109	1621	2133	2645	3157
5	**1024**	341	2389	4437	6485	8533	10581	12629

Alf's conjecture: the sum of [A1] and [A2] comprises the set of odd numbers. This has been verified up to 9999.

As p and n can take any value and the set of odd integers can be covered in the calculation of A then any sequence of numbers created using the Collatz conjecture can be replicated. This is done most conveniently by started at 1 and using the reverse sequence of numbers as a roadmap. More importantly all sequences that can be created by the Collatz conjecture approach can be created.

Rephrasing eqn 4:

Eqn 6. A1 = (B1*(2^2n)-1)/3 where B1 is a member of the set defined by [(6p + 1)] where p can take the values from 0 to infinity.

Table 4.

			6p + 1	A			
n	**2n**	**2^2n**	**B1**	**B1*(2^2n)**	**(B1*(2^2n)-1)/3**	**[6p + 1] ?**	**[6p – 1] ?**
1	2	4	1	4	1	0.00	0.33
2	4	16	1	16	5	0.67	1.00
3	6	64	1	64	21	3.33	3.67
4	8	256	1	256	85	14.00	14.33
5	10	1024	1	1024	341	56.67	57.00
6	12	4096	1	4096	1365	227.33	227.67
7	14	16384	1	16384	5461	910.00	910.33
8	16	65536	1	65536	21845	3640.67	3641.00
9	18	262144	1	262144	87381	14563.33	14563.67
10	20	1048576	1	1048576	349525	58254.00	58254.33
11	22	4194304	1	4194304	1398101	233016.67	233017.00
12	24	16777216	1	16777216	5592405	932067.33	932067.67

The calculated value for A has to be tested to see if it fits in the set [B1] or [B2] before it can be input as B1 or B2 for the next stage of calculation. When the value for A does not belong in either [B1] or [B2] then that branch can only continue by being multiplied by a power of 2.

Rephrasing eqn 5:

Eqn 7: A2 = (B2*(2^(2n+1)) - 1)/3 where B2 is a member of the set defined by [(6p - 1)] where p can take the values from 1 to infinity.

Table 5.

			6p - 1		A		
n	**2n + 1**	**2^(2n+ 1)**	**B2**	**B2*2^(2n+ 1)**	**(B2*2^(2n+ 1)-1)/3**	**[6p + 1] ?**	**[6p – 1] ?**
0	1	2	5	10	3	0.333333	0.666667
1	3	8	5	40	13	2	2.333333
2	5	32	5	160	53	8.666667	9
3	7	128	5	640	213	35.33333	35.66667
4	9	512	5	2560	853	142	142.3333
5	11	2048	5	10240	3413	568.6667	569
6	13	8192	5	40960	13653	2275.333	2275.667
7	15	32768	5	163840	54613	9102	9102.333
8	17	131072	5	655360	218453	36408.67	36409
9	19	524288	5	2621440	873813	145635.3	145635.7
10	21	2097152	5	10485760	3495253	582542	582542.3
11	23	8388608	5	41943040	13981013	2330168.667	2330169
12	25	33554432	5	167772160	55924053	9320675.333	9320675.667

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New sequences can be created at will starting from 1.

Once you have chosen a value for p then there are an infinite number of terms relating to powers of 2 that can be applied. So there are an infinite number of values of A that can be computed for each value of p. So there are an infinite possible number of branches at each value of p.

From the above I suggest (strongly) that the calculation process known as the Collatz conjecture will at its end point always equal 1 for all positive integer values.

Another conjecture: numbers in one branch above a fork point will not appear in any other branch above the fork point. Or alternatively: numbers only appear once on the integrated map of paths.

Alf Clark

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Appendix 1: Can A1 ever equal A2 ?

A1 = (B1*(2^2n)-1)/3

A2 = (B2*(2^2n+1)-1)/3

(B1*(2^2n)-1)/3 = (B2*(2^2n+1)-1)/3

Simplifying we get:

B1*(2^2n) = B2*(2^2n+1)

B1/B2 = (2^2n+1)/(2^2n)

B1/B2 = 2^[(2n+1)-2n] [just to be confusing the value for ''n'' in (2n+1) is not necessarily the same as the value for ''n'' in the term 2n.]

Simplifying [(2n+1)-2n] to k,

B1/B2 = 2^k (if k is negative then look at B2/B1 = 2^k)

2^k is even and an integer, B1 and B2 are odd

Therefore B1/B2 is fractional or odd therefore the proposition A1 = A2 is false.

Appendix 2: Can (6p + 1) = (6p – 1)

Now restating the above: can (6*p1 + 1) = (6*p2 – 1) : allows for the fact that p1 and p2 can take different values.

(6*p1 + 1) = (6*p2 – 1)

6*p1 - 6*p2 + 1 = - 1

6*(p1 – p2) = -2

p1 – p2 = -2/6

ie. p1 – p2 = -1/3

p1 and p2 are integers therefore the proposition (6p + 1) = (6p – 1) is false.

Appendix 3: a new analogy for Collatz Conjecture paths.

Consider a very large catchment for a river. At the periphery you have small streamlets that follow the topology and merge into larger streams. This process of streams merging continues and the streams eventually merge into a river. Now a drop of rain landing at the start of one of the streamlets will follow downhill always taking the path that leads to the river. However from the bank of the main river you can never tell which streamlet any particular drop of rain entered at nor the subsequent path it followed.

You can however follow the river upstream and with a 'map' find the appropriate streamlet. Or you can follow the flow upstream randomly taking left or right at a fork. You will eventually find a streamlet – not necessarily the streamlet that any particular drop of rain entered at.

So it is with the Collatz conjecture, only worse.

For every value of B1 or B2 there are an infinite number of possible paths. While you can deliberately construct a path leading from 1 the chance of accidently encountering any given number after a finite number of steps is infinitesimal. [all paths pass through 16, 8, 4, 2].

So it is possible to back track any sequence to a number but the chance of randomly creating that sequence is so small as to not be worth considering. However it is possible to create all paths starting at 1. You just need infinite time to examine all the solutions to find the one you are interested in.