Prime Prediction with Fibonacci Prime Vector triangles.

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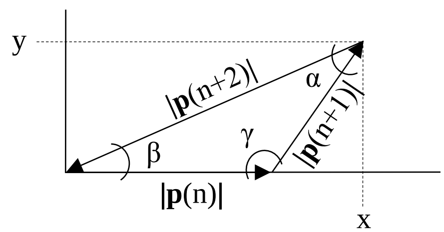
Introduction.

A method is given to estimate the position of the next prime based on the previous two primes. The method is constructed with: "Fibonacci" Prime Vector triangles. First the concept "Fibonacci" Prime Vector triangles will be explained. Then we will construct these triangles, analyze, make observations, and attempt to explain them. Finally, a method is presented to predict the location of the next prime based upon the 2 preceding prime numbers.

1. "Fibonacci" Prime Vector triangles.

Based on an internet topic on [1] the question was if there are patterns based on triangles with the side's length based on primes. Triangles where constructed as a vector summation of prime numbers:

$$\boldsymbol{p}(n+2) = \boldsymbol{p}(n+1) + \boldsymbol{p}(n) \tag{1}$$



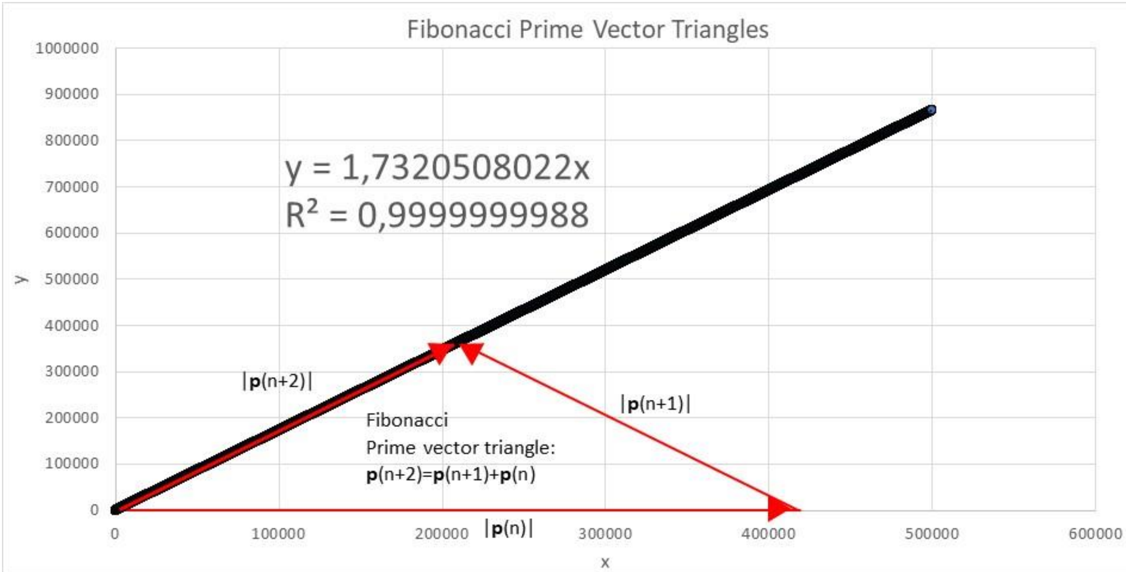
The coordinate x, y is determined by applying the law of cosines.

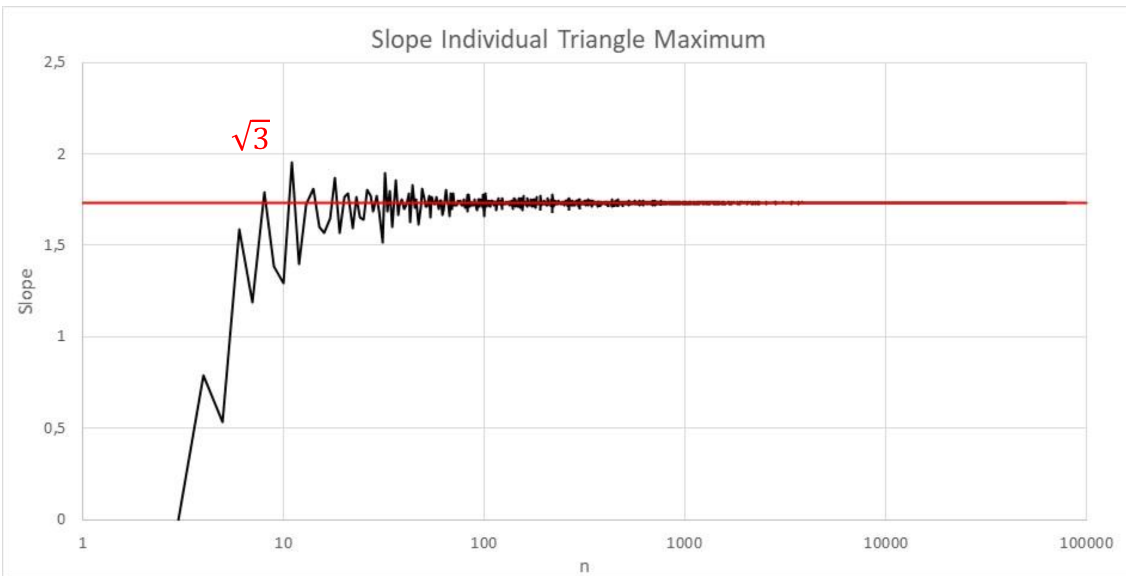
$$\begin{aligned}
\alpha &= cos^{-1}\left(\frac{|\boldsymbol{p}(n+1)|^2 + |\boldsymbol{p}(n+2)|^2 - |\boldsymbol{p}(n)|^2}{2|\boldsymbol{p}(n+1)||\boldsymbol{p}(n+2)|}\right)\\
\beta &= cos^{-1}\left(\frac{|\boldsymbol{p}(n)|^2 + |\boldsymbol{p}(n+2)|^2 - |\boldsymbol{p}(n+1)|^2}{2|\boldsymbol{p}(n)||\boldsymbol{p}(n+2)|}\right)\\
\gamma &= cos^{-1}\left(\frac{|\boldsymbol{p}(n)|^2 + |\boldsymbol{p}(n+1)|^2 - |\boldsymbol{p}(n+2)|^2}{2|\boldsymbol{p}(n)||\boldsymbol{p}(n+1)|}\right)\\
x &= |\boldsymbol{p}(n)| + |\boldsymbol{p}(n+1)| cos\,(\pi - \gamma)\\
y &= |\boldsymbol{p}(n+2)| cos\,\left(\frac{\pi}{2} - \beta\right)
\end{aligned} \tag{2}$$

The length of vector $|\boldsymbol{p}(n+2)|$ can be calculated with [2]:

$$|\boldsymbol{p}(n+2)| = |\boldsymbol{p}(n)|\cos(\beta) + \sqrt{|\boldsymbol{p}(n)|^2 cos^2(\beta) - |\boldsymbol{p}(n)|^2 + |\boldsymbol{p}(n+1)|^2} \tag{3}$$

The constructed triangles are from here on called: "Fibonacci" Prime Vector triangles. Next analysis is performed from the first 100000 primes the coordinates: x and y are determined.





It is observed that:

- On large scale the “Fibonacci” Prime Vector triangle height seems to grow linear.
- The slope y = 1,7320508022x of the line is remarkable close to: sqrt(3), 1,73205080756888.
- Remarkable: 2+3=5 is the only observed vector set where angular component is zero (no triangle but line).

The following questions arise:

- Do all “Fibonacci” Prime Vector triangles exist, are there combinations what are not possible?
- Where does the slope come from?

2. Extreme "Fibonacci" Prime Vector triangles.

Vectorset 2+3=5 is the only observed triangle with slope of 0. Do there exist more of these triangles with slope 0? To answer this question, we look at the parity of the prime numbers. Prime number 2 is the only even prime so: 2+odd=odd. For all other prime triangles with zero slope the following would be true: odd+odd=even, this expression does not hold while: all other primes are odd, there only exists one triangle with slope 0.

Do there exist any "Fibonacci" Prime Vector triangles what are not possible? If so, the following expression should hold:

$$|\boldsymbol{p}(n)| + |\boldsymbol{p}(n+1)| > |\boldsymbol{p}(n+2)| \tag{4}$$

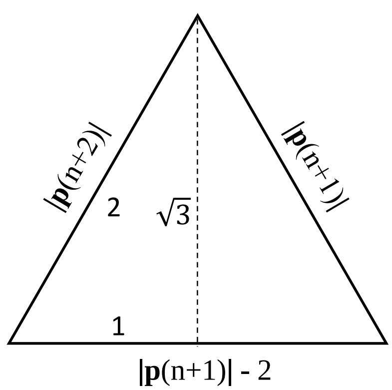
The sum (total length) of the preceding primes should be larger than the length of the vector $|\boldsymbol{p}(n+2)|$. The first question is: what is the biggest possible triangle? This should be a triangle where the preceding prime is a maximum. This occurs for twin prime numbers. Expression (4) will then become:

$$|\boldsymbol{p}(n+2)| < 2|\boldsymbol{p}(n+1)| - 2 \tag{5}$$

Expression (5) is known as: Bertrand–Chebyshev theorem [3]. We then can conclude that all "Fibonacci" Prime Vector triangles exist.

For twin primes the "Fibonacci" Prime Vector triangles are the biggest. This is for large prime numbers almost an equilateral triangle (difference 2 can be neglected for large prime numbers). An equilateral triangle has the following ratio: 1, 2, $\sqrt{3}$. The maximum slope for twin prime triangles then is: $\sqrt{3}$ like observed in the simulation.

It is remarkable that all other triangles (not twin prime) also tend to the slope of: $\sqrt{3}$. An explanation cannot be given other than this is a part of nature.



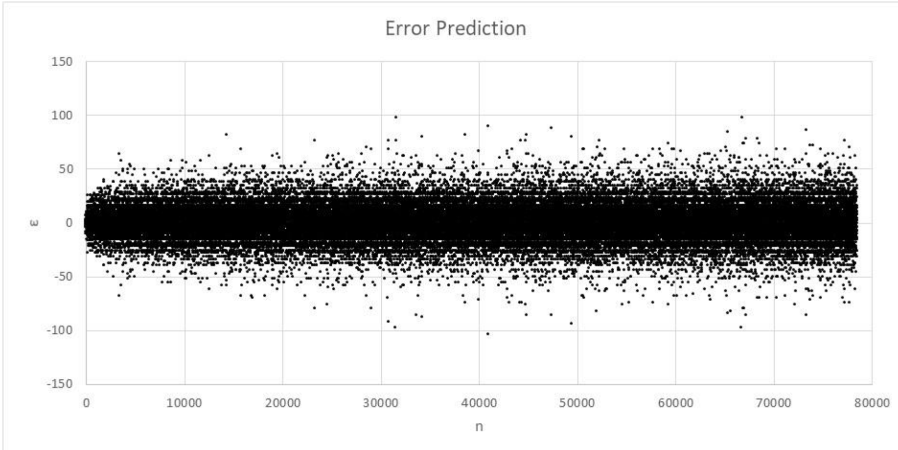
3. Prediction of next prime.

The slope for "Fibonacci" Prime Vector triangles is determined: $\sqrt{3}$ this corresponds to an angle of: $\beta = \frac{\pi}{3}$. With help of expression (3) we can predict the next prime. Expression (3) will become:

$$|\tilde{\boldsymbol{p}}(n+2)| = \frac{1}{2}|\boldsymbol{p}(n)| + \sqrt{-\frac{3}{4}|\boldsymbol{p}(n)|^2 + |\boldsymbol{p}(n+1)|^2} \tag{6}$$

We can determine the error between the prediction and the expected prime number:

$$\varepsilon(n+2) = |\tilde{\boldsymbol{p}}(n+2)| - |\boldsymbol{p}(n+2)| \tag{7}$$



For the first 80000 prime numbers the next prime number can be predicted with a maximum position inaccuracy between approximately: -100 and +100.

4. Summary.

All triangles based on the length of three following prime numbers exist. These triangles can be named: "Fibonacci" Prime Vector triangles. The height of these triangles will eventually grow with a maximum slope of: $\sqrt{3}$. This maximum slope is determined by twin primes. It is observed that all other triangles seem to follow this slope. This slope enables us to predict the position of the next prime based on the two preceding prime numbers.

5. Thoughts.

- The demonstrated method: prediction of prime numbers might explain the observed lines in the: Ulam Spiral.
- If there exists at least on prime number p for which: $n < p < 2n - 2$, Bertrands postulate [2] then there might exist infinate twin primes.

Any feedback is welcome:

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References.

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[3] Bertrand–Chebyshev theorem.
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https://en.wikipedia.org/wiki/Prime_gap#Upper_bounds