Proof of Goldbach conjecture

Abstract

The purpose of this paper is to prove Goldbach conjecture

by finding the number of Goldbach pairs

(All the prime numbers p1 less than or equal to N,

that are solutions to 2N=p1+p2)

and number of Non-Goldbach pairs

(All prime numbers p1 that are solutions to 2N=p1+c1,

where c1 is composite numbers) in the range from 6 to 2X.

Main proof

Let us consider the number of Goldbach pairs for all the even

numbers in the range from 6 to 2X.

Let us call this number for GB(2X) .

We have that

$$GB(2X) = \left(\sum_{p \geq 3}^{p_x} \pi(2X - p) - \pi(p - 2) \right)$$

where p is prime and p_x is the largest prime less than or equal to X

Explanation of the formula :

We can list the numbers of the form 2K=p1+p2, where K is any number less than or equal to X and p is an odd prime in the following way :

3+3, 3+5, 3+7, 3+11, 3+13...3+(2X-3)

5+5, 5+7, 5+11...5+(2X-5)

7+7,7+11, 7+13....7+(2X-7)

...

p_(x)+p_(x)....p_(x)+2X-p_(x)

As we can see from the following list,

this is equivalent to the formula :

$$GB(2X)=\left(\sum_{p\geq 3}^{p_x}\pi(2X-p)-\pi(p-2)\right)$$

Now consider the number of number pairs, C(2X), which are not Goldbach pairs, but the sum of a prime and a composite number in the range from 6 to 2X-3.

The number of these is equal to :

$$C(2X)=\frac{\left(\sum_{p_k\geq 3}^{p_x}(2X-p\ )-(p-2)\ \right)}{2}-GB(2X)$$

<u>Explanation of the formula </u>:

The reasoning behind this formula is that

we get all the numbers of the form 2N-p between 3 and 2N-3

regardless of whether they are prime or composite

and then subtract the number of Goldbach pairs.

Then we get C(2X).

We must divide the summation term with 2

since the numbers of the form 2N-p are all odd.

To find the number of Non-Goldbach pairs for the *single* value 2X

We subtract C(2*(X-1)) from C(2X)

which is :

$$C(2X)-C(2\cdot(X-1))$$

$$=\frac{\left(\sum_{p\geq 3}^{p_x}(2X-p)-(p-2)\right)}{2}-GB(2X)$$

$$-\left(\frac{\left(\sum_{p\geq 3}^{p_{x-1}}(2\cdot(X-1)-p)-(p-2)\right)}{2}-GB(2\cdot(X-1))\right)$$

$$=Q$$

In the case of a counter example to Goldbach conjecture, we must have

that Q=pi(2X-3)-1

Furthermore we must have that there are no Goldbach pairs

for the number 2X, so GB(2X)-GB(2*(X-1))=0

so GB(2X)=GB(2*(X-1))

So we should have if Goldbach conjecture is false that :

$$C(2X)-C(2\cdot(X-1))$$

$$=\frac{\left(\sum_{p\geq 3}^{p_x}(2X-p)-(p-2)\right)}{2}$$

$$-\left(\frac{\left(\sum_{p\geq 3}^{p_{x-1}}(2\cdot(X-1)-p)-(p-2)\right)}{2}\right)$$

$$=\pi(2X-3)-1$$

When P_x=P_(x-1)

Then the middle side of the equation

is equal to pi(X)-1

which can be easily seen when we look at a single prime value of p.

For any prime p, we have that :

(2X-p)-(p-2)/2-(2*(X-1)-p)-(p-2)/2

can be simply reduced to 1.

Since there are pi(X)-1 values of p, we can see that the middle side of the equation is equal to pi(X)-1 if P_x=P_(x-1)

If P_x is not equal to P_(x-1), which happens only if

P_x=X

then the left side of the equation is equal to

$pi(X)-1+(2X-X-(X-2))/2=pi(X)$

So $Q=pi(X)-1$ or $Q=pi(X)$

We can see that under these circumstances that

Q can clearly never be equal to $pi(2X-3)-1$

except for very small values of X.

That is the equations

$pi(X)-1=pi(2X-3)-1$

or $pi(X)=pi(2X-3)-1$

has finitely many solutions.

Because of this, we therefore have that Goldbach conjecture is true.

Q.E.D