The N-Queens Puzzle

Author: Alessandro Boatto

Abstract.

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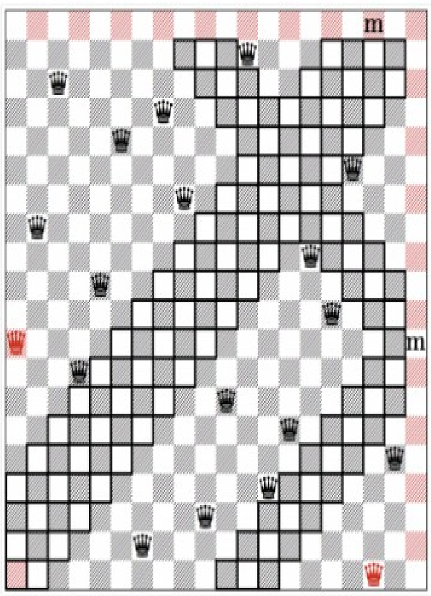
The 8-Queens Puzzle



A recent paper on the complexity of the *n*-Queens Completion Problem by researchers at the University of St Andrews may point the way to a new attack on one of the Millennium Prize Problems, the P vs NP problem. The paper is an exciting contribution to complexity theory, but it does not say that finding a correct solution to the 8-Queens puzzle or even to the *n*-Queens puzzle for all *n* would justify the award of the Millennium Prize.

As Ian Gent, one of the authors, comments: "The 8-Queens puzzle on the chessboard is a classic puzzle, and all solutions to it have long been known. It is also known that the more general *n*-Queens puzzle can be solved on all larger size chessboards: that is the puzzle of placing *n* queens on an *n*-by-*n* chessboard so that no queen is attacking another. The new research concerns the *n-Queens Completion Problem*, where not only is the board larger, but also some queens have already been placed. That is, if some queens have already been placed on the *n*-by-*n* board, can you find a solution to the *n*-Queens puzzle without moving any of those queens? The technical contribution claimed in this paper is that the *n*-Queens Completion Problem falls into the class known as *NP-Complete*. If correct, this means that any algorithm that can solve the *n*-Queens Completion Problem can be used indirectly to solve any other problem in the NP class. This does not apply to the original *n*-Queens puzzle, because the addition of pre-placed queens is critical.

"Unfortunately, some reports of our work have given the impression that solving the 8-queens puzzle, or the *n*-queens puzzle for all *n*, might result in the award of the Millennium Prize. This is not the case, for two reasons. First, as just mentioned, the paper is about the *n*-Queens Completion problem, not the original *n*-Queens puzzle. Second, even the discovery of an algorithmic solution to the *n*-Queens Completion puzzle for all *n* would not be enough. What would be necessary would be either a proof that there is an algorithm that can solve the *n*-Queens Completion puzzle in polynomial time, or a proof that no such algorithm exists."

Introduction.

- **Numerical order generation method.**
- **Method to understand the right numerical order to place the queens on the board.**

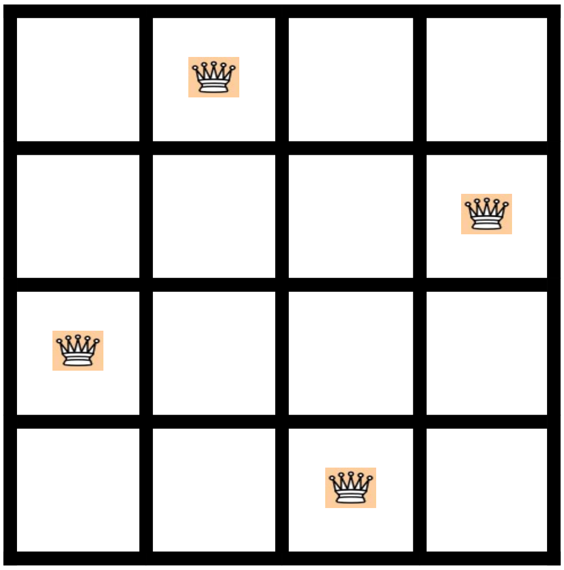


Fig.1

2 4 1 3

- The table consists of four columns **1 2 3 4**
- The columns are not numbered in an orderly way but in another order **2 4 1 3**
- This new numerical order of the columns is not random.
- The first column on the left with the number **2** indicates that the queen's position is in the second box.
- The second column with the number **4** indicates that the queen's position is in the fourth box.
- The third column with the number **1** indicates that the queen's position is in the first box.
- The fourth column with the number **3** indicates that the queen's position is in the third box.

Numerical order generation method.

The fundamental number is **1 2 3** and indicates a table with three columns and three rows and does not admit Unique and Distinct solutions, the minimum number of columns and rows that a table must have to admit Unique and Distinct solutions is four columns and four rows. (Fig.1)

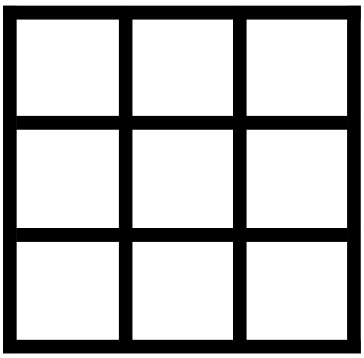


Fig. 2

1 2 3

The number **2 4 1 3** of (Fig.1) is generated by the possible combinations of the number **1 2 3** adding the number **4** to any combination but never before is after the number **3** (the number preceding the **4**).

- **1 2 3**
- **1 3 2**
- **2 1 3**
- **2 3 1**
- **3 1 2**
- **3 2 1**

Six combinations have been generated, and the number is added **4** . In this case each of the six combinations produces two combinations, at the end of the cycle there will be twelve combinations.

The first combination **1 2 3** it results:

- **4** 1 2 3
- 1 **4** 2 3

The second combination **1 3 2** it results:

- **4** 1 3 2
- 1 3 2 **4**

The third combination **2 1 3** it results:

- **4** 2 1 3
- 2 **4** 1 3

The fourth combination **2 3 1** it results:

- **4** 2 3 1
- 2 3 1 **4**

The fifth combination **3 1 2** it results:

- 3 1 **4** 2
- 3 1 2 **4**

The sixth combination **3 2 1** it results:

- 3 2 **4** 1
- 3 2 1 **4**

The generated combination or numerical order **2 4 1 3** of the table in Fig.1 results in the third combination **2 1 3** and it is also found in the fifth combination **3 1 2** with its numerical order in reverse **3 1 4 2** . In a table of four columns and four rows there are only one solutions while there are two distinct solutions as in this case (Fig. 1).

For the next table with five columns and five rows proceed by adding the number **5** to the twelve combinations just generated, always with the rule that the largest number in this case is the number **5** must not be placed before and after the number that precedes it by value thatin this case it is the number **4** . This time adding the number **5** for each combination three combinations will be generated, at the end of the cycle there will be thirty-six combinations.

- **4 1 2 3**
- **1 4 2 3**
- **4 1 3 2**
- **1 3 2 4**
- **4 2 1 3**
- **2 4 1 3**
- **4 2 3 1**
- **2 3 1 4**
- **3 1 4 2**
- **3 1 2 4**
- **3 2 4 1**
- **3 2 1 4**

The first combination **4 1 2 3** it results:

- 4 1 **5** 2 3
- 4 1 2 **5** 3
- 4 1 2 3 **5**

The second combination **1 4 2 3** it results:

- **5** 1 4 2 3
- 1 4 2 **5** 3
- 1 4 2 3 **5**

The third combination **4 1 3 2** it results:

- 4 1 **5** 3 2
- 4 1 3 **5** 2
- 4 1 3 2 **5**

The fourth combination **1 3 2 4** it results:

... .

... .

Method to understand the right numerical order to place the queens on the board.

The two main rules for understanding which combinations to discard are:

- Do not consider a combination with two consecutive numbers of value for example the combination **4 1 5 2 3** has the numbers **2 and 3** consecutive as is also truethe reverse is the combination **4 1 5 3 2** has the numbers **3 and 2 .**
- The second rule is understood by alternating the columns of the table for example the combination **4 1 2 3 5** and alternating **4 2 5** the number can be seen in (Fig. 3) **4** and the number **2** cross diagonally and the same thing can be seen for the numbers **1 and 3** (Fig.3).This is because they are only divided by one column and are only two units of difference **4-2 = 2** and **3-1 = 2**.

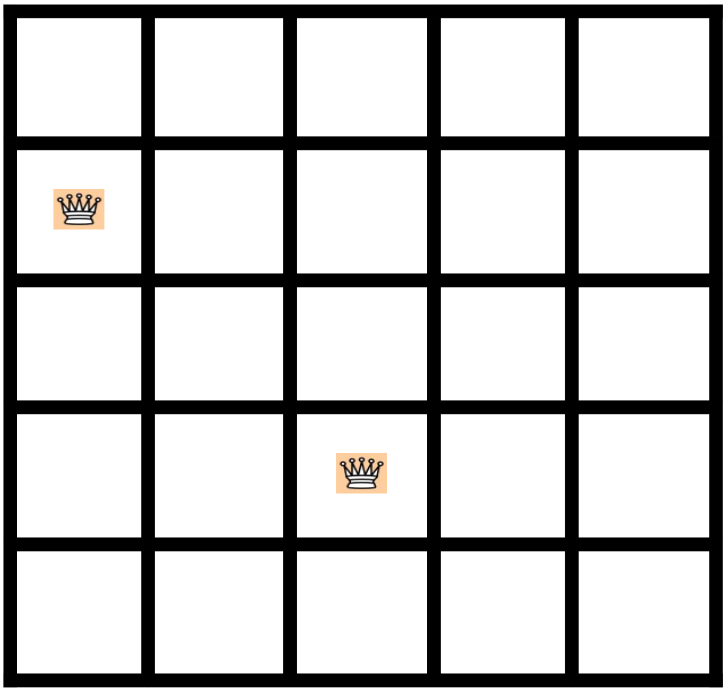


Fig.3

4 1 2 3 5

Acknowledgments.

20/02/2022 JESOLO Lido (VE)

Conclusions.

DEVELOP A PROGRAMSoftware and then VERIFY

References.

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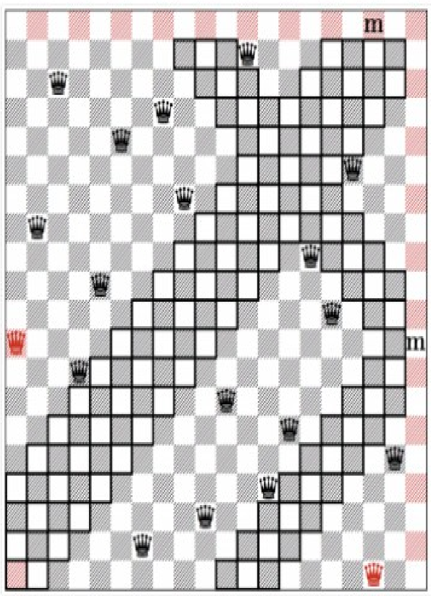

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Introduction.

- **Metodo di generazione ordine numerico.**
- **Metodo per comprendere il giusto ordine numerico per posizionare le regine sulla scacchiera.**

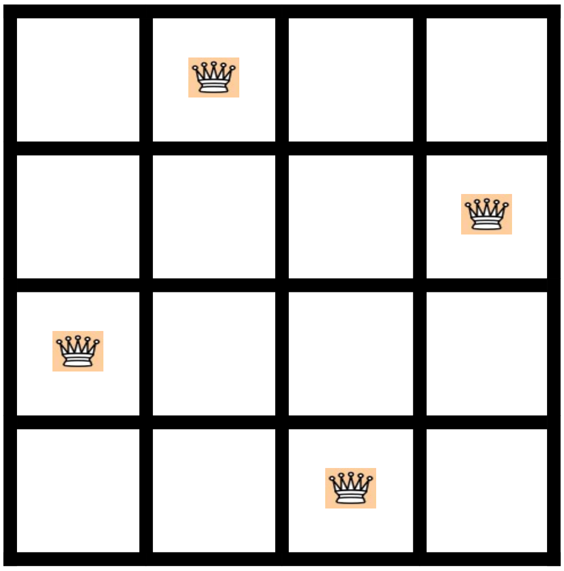

Fig.1

2 4 1 3

- La tabella è formata da quattro colonne **1 2 3 4**
- Le colonne non sono numerate in modo ordinato ma in un altro ordine **2 4 1 3**
- Questo nuovo ordine numerico delle colonne non è casuale .
- La prima colonna a sinistra con il numero **2** indica che la posizione della regina è nella seconda casella .
- La seconda colonna con il numero **4** indica che la posizione della regina è nella quarta casella .
- La terza colonna con il numero **1** indica che la posizione della regina è nella prima casella .
- La quarta colonna con il numero **3** indica che la posizione della regina è nella terza casella .

Metodo di generazione ordine numerico.

Il numero fondamentale è **1 2 3** e indica una tabella con tre colonne e tre righe e non ammette soluzioni Uniche e Distinte , il numero minimo di colonne e righe che una tabella deve avere per ammettere soluzioni Uniche e Distinte è quattro colonne e quattro righe . (Fig.1)

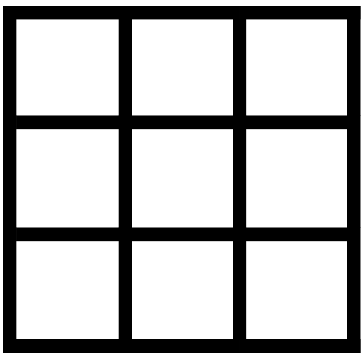

Fig.2

1 2 3

Il numero **2 4 1 3** della (Fig.1) è generato dalle combinazioni possibili del numero **1 2 3** aggiungendo il numero **4** ad ogni combinazione ma mai prima è dopo il numero **3** (il numero che precede il **4**) .

- **1 2 3**
- **1 3 2**
- **2 1 3**
- **2 3 1**
- **3 1 2**
- **3 2 1**

Si sono generate sei combinazioni , e si procede aggiungendo il numero **4** . In questo caso ogniuna delle sei combinazioni produce due combinazioni , alla fine del ciclo saranno dodici combinazioni .

La prima combinazione **1 2 3** risulta :

- <u>**4**</u> 1 2 3
- 1 <u>**4**</u> 2 3

La seconda combinazione **1 3 2** risulta :

- <u>**4**</u> 1 3 2
- 1 3 2 <u>**4**</u>

La terza combinazione **2 1 3** risulta :

- <u>**4**</u> 2 1 3
- 2 <u>**4**</u> 1 3

La quarta combinazione **2 3 1** risulta :

- <u>**4**</u> 2 3 1
- 2 3 1 <u>**4**</u>

La quinta combinazione **3 1 2** risulta :

- 3 1 <u>**4**</u> 2
- 3 1 2 <u>**4**</u>

La sesta combinazione **3 2 1** risulta :

- 3 2 <u>**4**</u> 1
- 3 2 1 <u>**4**</u>

<u>La combinazione generata o ordine numerico **2 4 1 3** della tabella in Fig.1 risulta nella terza combinazione **2 1 3** e si trova anche nella quinta combinacione **3 1 2** con il suo ordine numerico al contrario **3 1 4 2** .</u> In una tabella di quattro colonne e quattro righe le soluzioni Uniche sono solo una mentre le soluzioni distinte sono due come in questo caso (Fig.1) .
Per la prossima tabella con cinque colonne e cinque righe si procede aggiungendo il numero **5** alle dodici combinazioni appena generate sempre con la regola che il numero più grande che in questo caso è il numero **5** non deve essere posizionato prima e dopo il numero che lo precede di valore che

in questo caso è il numero **4**. Questa volta aggiungendo il numero **5** ad ogniuna combinazione verranno generate tre combinazioni , alla fine del ciclo saranno trentasei combinazioni .

- **4 1 2 3**
- **1 4 2 3**
- **4 1 3 2**
- **1 3 2 4**
- **4 2 1 3**
- **2 4 1 3**
- **4 2 3 1**
- **2 3 1 4**
- **3 1 4 2**
- **3 1 2 4**
- **3 2 4 1**
- **3 2 1 4**

La prima combinazione **4 1 2 3** risulta :

- 4 1 **5** 2 3
- 4 1 2 **5** 3
- 4 1 2 3 **5**

La seconda combinazione **1 4 2 3** risulta :

- **5** 1 4 2 3
- 1 4 2 **5** 3
- 1 4 2 3 **5**

La terza combinazione **4 1 3 2** risulta :

- 4 1 **5** 3 2
- 4 1 3 **5** 2
- 4 1 3 2 **5**

La quarta combinazione **1 3 2 4** risulta :

…......................................

…............................

Metodo per comprendere il giusto ordine numerico per posizionare le regine sulla scacchiera.

Le due regole principali per capire quali sono le combinazioni da scartare sono :

- Non prendere in considerazione una combinazione con due numeri consecutivi di valore per esempio la combinazione **4 1 5 2 3** ha i numeri **2** e **3** consecutivi come vale anche il contrario la combinazione **4 1 5 3 2** ha i numeri **3** e **2 .**
- La seconda regola si comprende alternando le colonne della tabella per esempio la combinazione **4 1 2 3 5** e alternando **4 2 5** si può notare in (Fig.3) il numero **4** e il numero **2** si incrocino in diagonale e la stessa cosa si può notare per i numeri **1** e **3** (Fig.3) . Questo perchè sono divisi solo da una colonna e sono solo due unità di differenza **4-2 = 2 e 3-1 = 2** .

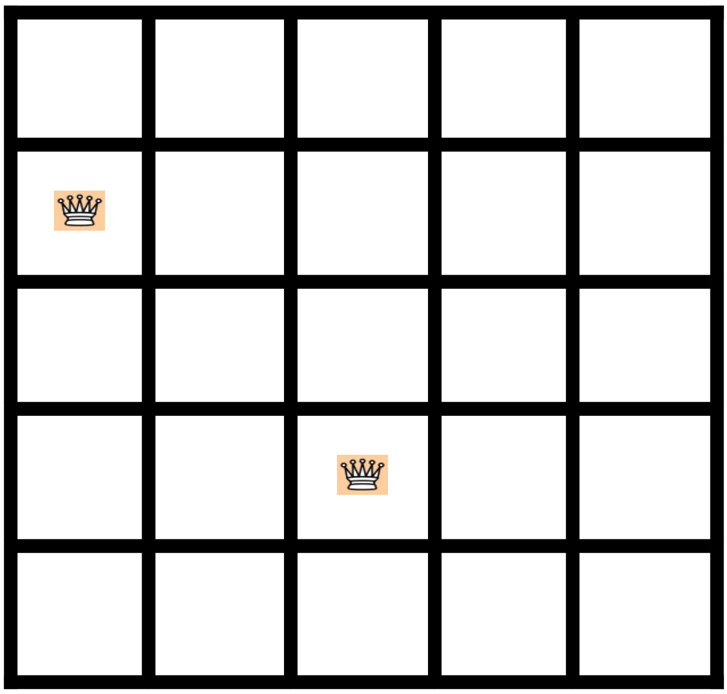

Fig.3

4 1 2 3 5

Acknowledgements.

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Conclusions.

SVILUPPARE UN PROGRAMMA Software e poi VERIFICARE

References.

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