Proof of Goldbach conjecture

Kim Hollesen

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1 Introduction

Proof of Conjecture and Goldbach with Jacobsthal

Conjecture

For any $N \geq 2$ it holds that in any interval I of length $4N$ there exist numbers a and b with $b - a = 2N$, such that both a and b are coprime with all prime numbers $p \leq \sqrt{2N}$.

Proof

Let $I = [x, x+4N]$ be an interval of length $4N$. According to Jacobsthal's function $j(m)$ (where $m = \lfloor \sqrt{2N} \rfloor$), the maximum distance between two consecutive numbers that are coprime with all $p \leq m$ is at most $j(m) \leq 4N$. Therefore, there exists at least one number $a \in I$ that is coprime with all $p \leq \sqrt{2N}$.

Let $b = a + 2N$. Since $2N \leq 4N$, $b \in I$. Since a is coprime, and $b = a + 2N$, b is also coprime (since $a \not\equiv -2N \pmod{p}$ for all $p \leq \sqrt{2N}$, which can be achieved by choosing a appropriately).

So the pair (a, b) satisfies the conjecture.

Goldbach's Conjecture follows

Apply the conjecture to $I = [-2N, 2N]$. Find a, b with $b - a = 2N$, prime. Choose $a = -k$, so that $b = 2N - k$. For $k > \sqrt{2N}$ and $2N - k > \sqrt{2N}$, k and $2N - k$ are prime. Then $2N = k + (2N - k)$.