

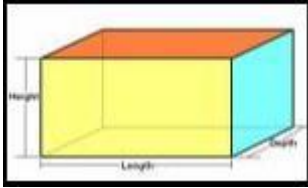
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### Perfect Cuboid

[www.unsignedproblems.org](http://www.unsignedproblems.org)

An Euler Brick is just a cuboid, or a rectangular box, in which all of the edges (length, depth, and height) have integer dimensions; and in which the diagonals on all three sides are also integers.



So if the length, depth and height are  $a$ ,  $b$ , and  $c$  respectively, then  $a$ ,  $b$ , and  $c$  are integers, as are the quantities  $\sqrt{a^2+b^2}$  and  $\sqrt{b^2+c^2}$  and  $\sqrt{c^2+a^2}$ .

*The problem is to find a perfect cuboid, which is an Euler Brick in which the space diagonal, that is, the distance from any corner to its opposite corner, given by the formula  $\sqrt{a^2+b^2+c^2}$ , is also an integer, or prove that such a cuboid cannot exist.*

### Solution

There is no rational solution to the perfect cuboid. It is the radius of a sphere. (push it back from the edge to the centre radius of the sphere)

$$\text{So... } a^2 + b^2 + c^2 = d^2$$

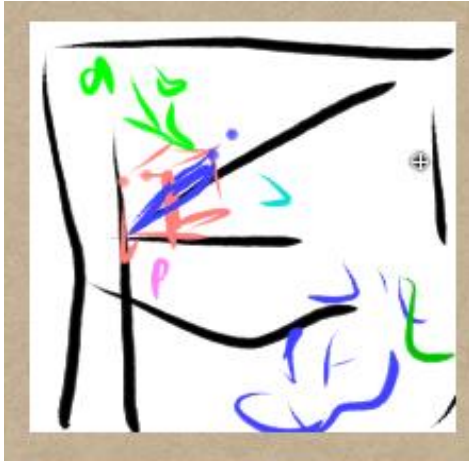
At the 45° centre core set of lines (4 of (2 spheres))

$$a = b = c \text{ therefore}$$

$$3a^2 = d^2$$

Implying positively that there is not a solution...

(because when you root both sides, as root 3 is not a rational, then both  $a$  and  $d$  cannot be rational)



$$a^2 + b^2 + c^2 = d^2$$

$$(a + s_1)^2 + (b + s_2)^2 + (c + s_3)^2 = d^2$$

$a^2 + b^2 + c^2 + s_1^2 + s_2^2 + s_3^2 = d^2 - 2as_1 - 2bs_2 - 2cs_3$   
and they are all rational except  $d$ .. so there is conclusively no perfect  
rectangloid...