Beal's Conjecture & Its Rigorous Proof

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Abstract – This paper deals with Beal's conjecture & its proof. With the discovery of Beal's conjecture in1993 and declaration of prize for a person who gives a proof to Beal's Conjecture or disprove it or search a counter example to it. A number of mathematicians attempted to attack the problem.

In the first group mathematicians like Peter Norvig and Edword D. Collins, Akhil Verma etc. focused to search counter examples using super computers and advanced programs with very high values of A, B, C and x ,y ,z but failed. In the second group mathematicians like James Constant, Don Blays, and others tried to disprove Beal's conjecture and failed. In the third group mathematician like Dr. Raj C. Thagraj etc. attempted to prove the Beal's Conjecture but their proof could not be recognized by mathematical society of America, so far.

This paper comes to third category and attempts to prove the Beal's Conjecture in a logical manner. It shows that a solution to $A^x+B^y=C^z$ can exist if and only if A,B,C have a common prime factor r or product of two or more prime numbers, for every A, B,C positive integers, and exponents x, y, z > 2 and positive integers. This paper provides a method to generate Beal's equation and to calculate common factor r. This proof also gives a way to prove Fermat's Last Theorem (as will be given later).

Introduction – In the present proof we have used properties of positive integers and rules of indices (ie exponents) as mentioned below.;

(I) for every positive integer a, a^x is also a positive integer,
(II) for positive integers a and b, $a^x+b^y=d$ is also a positive integer,

With these simple postulates we have derived common factor r. We have shown that existence of r (i.e. a common factor) in A, B, C is necessary and sufficient condition for existence of a solution to Beal's equation.

Statement of Beal's equation :- If A, B, C are positive integers (used as bases) and x, y, z >2 are positive integers (used as exponents) then the Beal's equation

$A^x+B^y=C^z$ ……………………………………………………………...….(1)

Can have a solution if and only if A, B, C have a common prime factor. **

<u>Proof:- Part I</u>.

Firstly we assume that A,B,C in Beal's equation have a common factor r , **i.e. greatest common divisor of A, B, C = r and Beal's Conjecture is true**

Therefore we can express A,B,C in following way:

$A= ar$ ……………………………………………………..….….………(2a)

$B= br$ ……………………………………………………..….…………(2b)

$C= cr$ ……………………………………………………..….………..(2c)

Where a,b,c are relatively prime.

Now substituting values of A,B,C from equation (2) in equation (1) we get

$(ar)^x+(br)^y=(cr)^z$

or $a^xr^x+b^yr^y=c^zr^z$

Dividing both sides with r^x, we get

$\mathbf{a^x+b^y*r^{(y-x)}=c^zr^{(z-x)}}$ …………………………...…………………………(3)

Here we find that second term $b^yr^{(y-x)}$ cannot be expressed as power of single term. Similarly third term $c^zr^{(z-x)}$ cannot be expressed as a power of single term.

Illustration:- 5^7*2^7=(5*2)^7=10^7

But 5^7*2^4 is not equal to (5*2)^7 or (5*2)^4

Therefore (5^7)*(2^4) can not be expressed as (5*2)^7

So $b^yr^{(y-x)}$ cannot be expressed as (b*r)^y .

Hence we can conclude that equation (3) is not in the form of Beal's equation.

<u>In other words we can say that existence of common factor r is necessary condition to $A^x+B^y=C^z$ have a solution.</u>

<u>Part II</u>. Here we start with two arbitrary positive **integers a,b** and two positive integer exponents x,y>2.

We know that for every positive integer a ,a^x is also positive integer, and for every a,b positive integers, $a^x+b^y=d$ ……………..………………………………..(4)

Is also a positive integer.

Now we multiply both side of equation (4) by a multiplication factor **R**=$d^{[lcm\ (x,\ y).]}$

We can evaluate **R** in the following way

Let x=m*p, y=m*q

Therefore greatest common divisor of x,y=gcd(x,y)=m ……………..…….(5)

And lowest common multiple of x, y = lcm (x,y)= mpq …....……….…….(6)

And multiplication factor **R**=$d^{[lcm(x,y)]}=d^{(mpq)}=d^{(xq)}=d^{(py)}$ …..………………….(7)

And common factor r=d

So by multiplying equation (4) on both sides by **R**= $d^{(mpq)}$ we get

$[a^x+b^y=d]*d^{(mpq)}$

or $a^x*d^{(mpq)}+b^y*d^{(mpq)}=d^{(mpq+1)}$

or $a^x*d^{(xq)}+b^y*d^{(py)}=d^{(mpq+1)}$

or $[a*d^q]^x+[b*d^p]^y=d^{[1+lcm(x,y)]}$

or $A^x+B^y=d^{[1+lcm(x,y)]}=C^z$ …………………………………….………..(8)

Where $A=a*d^q$,$B=b*d^p$,C=d and z=[1+lcm(x,y)] …………...……………(9)

Equation no.(8) is Beal's equation, and this has been generated from the general equation no. (4) which is not a Beal's equation.

Therefore existence of a common factor **r=d=(a^x+b^y)** is sufficient condition to solve a Beal's Equation. Here it should be noted that $a^x + b^y = d$ is always greater or equal to 2 **for every positive integers a, b, x, y.**

Hence proved

Method to generate Beal's Equation:

S. N.	a	b	x	y	a^x	b^y	$d=a^x+b^y$	Lcm (x,y)	Multiplication Factor= d[lcm(x,y)]	Beal's equation $(a^x+b^y=d)*d^{lcm(x,y)}$	Final Beal's equation
1	1	1	3	3	1	1	$1^3+1^3=2$	3	2^3	$[1^3+1^3=2]*2^3$	$2^3+2^3=2^4$
2	1	1	4	4	1	1	$1^4+1^4=2$	4	2^4	$[1^4+1^4=2]*2^4$	$2^4+2^4=2^5$
3	1	1	5	5	1	1	$1^5+1^5=2$	5	2^5	$[1^5+1^5=2]*2^5$	$2^5+2^5=2^6$
4	1	1	6	6	1	1	$1^6+1^6=2$	6	2^6	$[1^6+1^6=2]*2^6$	$2^6+2^6=2^7$
5	1	1	7	7	1	1	$1^7+1^7=2$	7	2^7	$[1^7+1^7=2]*2^7$	$2^7+2^7=2^8$
6	1	1	8	8	1	1	$1^8+1^8=2$	8	2^8	$[1^8+1^8=2]*2^8$	$2^8+2^8=2^9$
7	1	1	9	9	1	1	$1^9+1^9=2$	9	2^9	$[1^9+1^9=2]*2^9$	$2^9+2^9=2^{10}$
8	1	1	10	10	1	1	$1^{10}+1^{10}=2$	10	2^{10}	$[1^{10}+1^{10}=2]*2^{10}$	$2^{10}+2^{10}=2^{11}$
9	1	1	11	11	1	1	$1^{11}+1^{11}=2$	11	2^{11}	$[1^{11}+1^{11}=2]*2^{11}$	$2^{11}+2^{11}=2^{12}$
10	1	1	12	12	1	1	$1^{12}+1^{12}=2$	12	2^{12}	$[1^{12}+1^{12}=2]*2^{12}$	$2^{12}+2^{12}=2^{13}$
11	1	1	13	13	1	1	$1^{13}+1^{13}=2$	13	2^{13}	$[1^{13}+1^{13}=2]*2^{13}$	$2^{13}+2^{13}=2^{14}$
12	1	2	3	3	1	8	$1^3+2^3=9$	3	9^3	$[1^3+2^3=9]*9^3$	$9^3+18^3=9^4$
13	1	2	4	4	1	16	$1^4+2^4=17$	4	17^4	$[1^4+2^4=17]*17^4$	$17^4+34^4=17^5$
14	1	2	5	5	1	32	$1^5+2^5=33$	5	33^5	$[1^5+2^5=33]*33^5$	$33^5+66^5=33^6$
15	1	2	6	6	1	64	$1^6+2^6=65$	6	65^6	$[1^6+2^6=65]*65^6$	$65^6+130^6=65^7$
16	1	2	7	7	1	128	$1^7+2^7=129$	7	129^7	$[1^7+2^7=129]*129^7$	$129^7+258^7=129^8$
17	2	2	3	3	8	8	$2^3+2^3=16$	3	16^3	$[2^3+2^3=16]*16^3$	$32^3+32^3=16^4$
18	2	3	3	3	8	27	$2^3+3^3=35$	3	35^3	$[2^3+3^3=35]*35^3$	$70^3+105^3=35^4$
19	3	3	3	3	27	27	$3^3+3^3=54$	3	54^3	$[3^3+3^3=54]*54^3$	$162^3+162^3=54^4$
20	3	4	3	3	27	64	$3^3+4^3=91$	3	91^3	$[3^3+4^3=91]*91^3$	$273^3+364^3=91^4$
21	3	5	3	3	27	125	$3^3+5^3=152$	3	152^3	$[3^3+5^3=152]*152^3$	$456^3+760^3=152^4$
22	1	2	3	4	1	16	$1^3+2^4=17$	12	17^{12}	$[1^3+2^4=17]*17^{12}$	$(83521)^3+(9826)^4=171^3$
23	1	2	3	5	1	32	$1^3+2^5=33$	15	33^{15}	$[1^3+2^5=33]*33^{15}$	$(39135303)^3+(71874)^5=33^{16}$

24	1	2	3	6	1	64	$1^3+2^6=65$	6	65^6	$[1^3+2^6=65]*65^6$	$(4225)^3+(130)^6=65^7$
25	1	2	3	9	1	256	$1^3+2^9=257$	9	257^9	$[1^3+2^9=257]*257^9$	$(16974593)^3+(514)^6=(257)^7$
26	2	2	3	4	8	16	$2^3+2^4=24$	12	24^{12}	$[2^3+2^4=24]*24^{12}$	$(331776)^3+(27648)^4=(24)^{13}$
27	2	3	3	4	8	81	$2^3+3^4=89$	12	89^{12}	$[2^3+3^4=89]*89^{12}$	$(125484482)^3+(2114907)^4=(89)^{13}$
28	3	4	3	5	27	1024	$3^3+4^5=1051$	15	1051^{15}	$[3^3+4^5=1051]1051^{15}$	$(384700156299753)^3+(4643742604)^5=(1051)^{16}$
29	3	4	3	6	27	4096	$3^3+4^6=4123$	6	4123^6	$[3^3+4^6=4123]*4123^6$	$(50997387)^3+(16492)^6=(4123)^7$
30	3	4	4	6	81	4096	$3^4+4^6=4177$	12	4177^{12}	$[3^4+4^6=4177]4177^{12}$	$(218632479699)^4+(69789316)^6=(4177)^{13}$

Note:- In equation no. 12 a=1,b=2,x=y=3,d=$1^3+2^3=9$ can also be written as $e^2=3^2$. Therefore multiplication factor can be reduced to $d*e^3=9*3^3=3^5$. Now we will get a smaller Beal's equation $(1^3+2^3=9)*3^3$ or $(3^3+6^3=3^5)$

So:-

S.N.	a	b	x	y	a^x	b^y	$d=a^x+b^y$	Lcm (x,y)	Multiplication Factor= $e^{[lcm(x,y)]}$	Beal's equation $(a^x+b^y=d)*d^{lcm(x,y)}$	Final Beal's equation
12a	1	2	3	3	1	8	$1^3+2^3=9=(e^2=3^2)$	3	$e^3=3^3$	$[1^3+2^3=9]*3^3$	$3^3+6^3=3^5$

Characteristic Properties Of Beal's Coefficients A,B,C

(I) If A and B both are even then C will also be even

(II) If A and B both are odd then also C will be even

(III) If A is even and B is odd or vice versa then C will be odd

(IV) All the three bases A,B,C used in Beal's equation are composite number having common prime factor or common product of two or more prime numbers

(V) If powers of A and B are same (i.e. x=y=n) then the power of third term C will be z>=(n+1),therefore for x,y,z>2 and A,B,C obeying Beal's equation then $A^n+B^n=C^n$ is always impossible. This is Fermat's last Theorem for composite numbers

(VI) If A,B,C are co primes and exponents x,y,z obey the condition (1/x+1/y+1/z<=1)then $(A^x+B^y=C^z)$ has only finite many solutions within the set of positive integers. This is called Fermat-Catalon conjecture.Only ten solutions are known upto now and every solution has at least one exponent 2.