The Goldbach conjectures

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Abstract

We deal with two problems known by the name of Goldbach conjectures, weak and strong versions.

The Goldbach conjectures

They are two of the oldest and best-known problems of number theory and all mathematics. The first, known as the weak one, states that an odd number greater than 25 is always the sum of three prime numbers, when the strong Goldbach conjecture states that an even number greater than 22 is always the sum of two prime numbers. This last one has been shown to hold up through 4.10^{28} but remains unproven despite considerable effort. We present here a very elementary approach of the problems and prove them by the same way. For the strong version, we present two approaches !

The weak Goldbach conjecture

First of all, an odd number can be different of the sum of three powers of odds.

We have 2k=4p+2m+1+2k'-1 that describes all even numbers from 4p+2k' until infinity when m describes all the integers.

Thus 2k+1=4p+2m+1+2k' describes all odd numbers from 4p+2k'+1 until infinity for the same m.

But 3p+2m describes all the odd numbers from 3p to infinity when m describes all the integers.

Hence, there always exists m' for which 3p+2m'=q is a prime number.

We deduce that 2k+1=p+3p+2m'+2(m-m')+2k'+1=p+q+2(m-m')+2k'+1

Describes all the odd numbers from 4p+2k'+1 until infinity and this for every k'.

But 2(m-m')+2k'+1 describes all the odd numbers from 2(m-m')+1 to infinty when k' describes all the integers.

There always exists k'=k'' for which 2(m-m')+2k''+1=r a prime number.

In conclusion :

2k+1=p+q+r which describes the odds from 4p+2k'+1 to infinty is always the sum of three prime numbers.

We want to calculate the first value of 2k+1 :

Let p=5 then 5+17+3=2k+1 means k=12=2p+k'=10+k' and we know now that the conjecture is true for all the odds from 25 to infinity !

The strong Goldbach conjecture

First approach

First of all, we must notice that an even number is not always the sum of two powers of odd numbers. A counter example will show it : n>1, m=1 (or m=2 or 3) and the exponent n=2 n=2

$$6 = p_1^{\ 2} + p_2^{\ 2}$$

$$6 - 2p_1p_2 = p_1^{\ 2} + p_2^{\ 2} - 2p_1p_2 \geq 0$$
$$3 \geq p_1p_2$$
$$p_1 = 1$$

And for

$$n > 2$$
$$6 - 2p_1p_2 = p_1^{\ n} + p_2^{\ n} - 2p_1p_2 > p_1^{\ 2} + p_2^{\ 2} - 2p_1p_2 \geq 0$$
$$3 \geq p_1p_2$$
$$p_1 = 1$$

There is an even number (6) which is not equal to the sum of the powers of two odd numbers ! It means that, in what follows, n=1.

For every even number 2m greater or equal to 12, there exists an integer I, a prime p and an integer k for which :

$$2m = 4ip + 2k$$

We want to suppose there exists an even which is not equal to the sum of two primes. We must have

$$\frac{q + r - 2k}{4p} \notin N$$

With q and r are primes. Thus it is rational :

$$\frac{q + r - 2k}{4p} = \frac{a}{b}$$

With GCD(a,b)=1

But

$$4ap = b(q + r - 2k)$$

b does not divise a, and as p is prime, b does not divise p, hence it divises 4, consequently b=4 or b=2.

If b=4

$$ap = q + r - 2k$$

It means a even : impossible because GCD(a,b)=1

Thus b=2

$$\frac{q + r - 2k}{2p} = a$$

As i can take every value : let i=a

$$2m = 4ip + 2k = 4ap + 2k = 2\left(q + r\right) - 2k$$

Or

$$m = 2ip + k = 2ap + k = q + r - k$$

And

$$\frac{q + r - 2k}{2p} = a = i = (2k' + 1)w$$

$$k', w \in N$$

As

$$2m = 4ip + 2k = q + r + 2\left(k'' - k\right)$$

$$w = 2ip - \left(k'' - k\right)$$

It means

$$\frac{q}{p} + \frac{r}{p} = 2\left(w - 2i\right) + \frac{2n}{p}$$

Where w is integer or

$$\frac{q}{p} + \frac{r}{p} = 2w + \frac{2k}{p}$$

Let

$$w = \frac{i}{2k' + 1}$$

And

$$\frac{q}{p} + \frac{r}{p} = 2(w - 2i) + \frac{2n}{p}$$

$$= -\frac{2(4k'+1)i}{2k'+1} + \frac{2m}{p} = -\frac{(4k'+1)m - (4k'+1)k}{(2k'+1)p} + \frac{2m}{p} = \frac{m + (4k'+1)k}{(2k'+1)p}$$ Thus

$$m + \left(4k' + 1\right)k = \left(2k' + 1\right)\left(q + r\right) = \left(q + r - k\right) + \left(4k' + 1\right)k = q + r + 4k'k$$

Hence

donc

$$2k = q + r$$

Or

$$2m = 2(q + r) - 2k = q + r$$

But we have supposed the contrary : it means that it is impossible to suppose that there exists an even is different of the sum of two primes ! Remark that q and r can be only . We conclude that an even is always the sum of two primes !

Second approach

Also 2k=4p+2m+2k' describes all the even numbers from 4p+2k' to infinity when m describes all the integers and, we saw it, there always exists m' and q a prime number for which 3p+2m'=q.

Thus 2k=p+q+2(m-m')+2k' describes all the evens from 4p+2k' to infinity and this for every k'.

Particularly, there always exists k'=k'' for which : 2k'+2(m-m')=0 and

2k=p+q describes all the evens from 4p+2k' to infinty and is the sum of two primes.

Practically :

For p=5 and 5+17=2k or k=11=2p+k'=10+k'.

It means that the strong conjecture is true for all the evens from 22 to infinity !

Conclusion

Our approach was sufficient to demonstrate the Goldbach conjectures.

Bibliography

[1] Fliegel, Henry F.; Robertson, Douglas S. (1989). "Goldbach's Comet: the numbers related to Goldbach's Conjecture". *Journal of Recreational Mathematics* **21** (1): 1–7.

[2] Chen, J. R. (1973). "On the representation of a larger even integer as the sum of a prime and the product of at most two primes". *Sci. Sinica* **16**: 157–176.