■ A new Solution To Collatz, (3n + 1) conjecture ■

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■ Collatz conjecture and Statement of problem ■

One of the more well - known unsolved problem in number theory is the Collatz, (3n + 1) conjecture, the conjecture asserts that starting with any given number n, do the following:

If n is even, then divided by 2.

If n is odd, then multiply by3 and add 1. Now take the result and repeat the procedure. So far, every time people have tried this, they have always found that eventually they reach the number 1. And people have tried it on trillions of numbers so far. Currently, the conjecture has been verified for all numbers up to 5.6×10^{13}.

■ STATMENT OF PROBLEM ■

Does there exist any number for which the procedure does not eventually lead to 1?

● Basic concepts of the collatz, (3n + 1) conjecture ●

During the first 1960's various problem related to the 3n + 1 appear in print, typically as unsolved problem. This includes one of the original problem of Collatz from the 19 30's, which concerned the behavior under iteration of the function:

$$U=(2n)=3n, U=(4n+1)=3n+1, U=(4n+3)=3n+2$$

The function U (n) defines a permutations of the integers, and the question concerns whether the iterates of the value n = 8 form an infinite set. This problem was raised by by Murray llama in 1963 and remain unsolved.Since 3n + 1 is even if n is odd, it is convenient to work with the modified Collatz function:

$$T(n) = \begin{cases} 3n + 1 / 2, & \text{if } n=1 \pmod 2 \\ n/2, & \text{if } n=0 \pmod 2. \end{cases}$$

CASE - I

Firstly, any natural numbers can be written in different ways. But for the purpose of proof of the Collatz, (3n + 1) conjecture, I assign a three function form.i.e ¥n € / N,

n	f1 = 3n -2	f2 = 3n -1	f3 = 3n
1.	1	2	3
2.	4	5	6
3.	7	8	9
:	:	:	:....etc.

But, I believe that for the purpose of elimination number 1 from f1 = 3n -2, simply add 3 on f1 = 3n -2, then you can get a new function form (i. e the collatz, (3n + 1) conjecture).

Therefore, we can easily understand the relationship between the function f1 = 3n -2 and the Collatz, (3n + 1) conjecture.

n.	f1 = 3n -2	C (n) = 3n + 1
1.	1	4
2.	4	7
3.	7	10
:	:	:...etc.

Note:-

I) If n is odd, then f1 is Odd and C (n) is Even.

II)If n is even, then f1 is Even and C (n) is Odd.

This implies that C (n)=(3n + 1)is an elements of the function f1 =(3n -2).

■ CASE - II

Secondly, I assign two different function form for all Odd, positive integers to indicate that the relationship between the two pairs of the twin prime numbers change functional value place and the $C(n) = 3n+1$ conjecture. i.e ¥m, n € / N and n € { n1, n2 }; and

$C(n) = 3n + 1,$

$f=(n1)=C(n1)/2^2 =3(n1)+1/2^2=3m-2$

$f(n2)= C(n2)/2^1=3(n2)+1/2 =6m-1$

where $n1 = 4m-3$ and $n2 = 4m-1$. look the following illustrations table 1.3 below.

n1	n2	C (n)	f (n1)	f(n2)
1		4	1	
	3	10		5
5		16	4	
	7	22		11
9		28	7	
	11	34		17
:	:	:	:	:. ..etc.

● Note:-

From the above table 1.3, we can easily observe that:

I)5 € n1,but 5 € f (n2); 7 € n2, but 7 € f (n1).

II)11 € n2, and 11€f (n2); 13 € n1, and 13 € f (n1).

In general,add the common difference d= 12 to get the next half of the two pairs of twin prime numbers that occurs place change wrt the function results.(i.e (5,7), (17,19),(29, 31), (41,43,).......etc.)

Conversly,the next half of the two pairs of twin prime numbers that don't occur place changes with the function results.
i. e {(11,13), (23,25),(35,37), (47,49)..etc.}

Case - III

Finally, I split $n1=(4m-3)$ into two forms and I assign $n1 = 8m-7$ and $n3^* = 8m-3$ to indicate the Collatz,(3n + 1) conjecture turning point appear's at $n3^* = 8m-3$ and I call it " ladders".

¥ i, m € / N,thus

If $n1 = 8m-7$, then $f(n1) = C(n1)/4$

$\Rightarrow f(n1) = 3(n1) + 1 / 2 \wedge 2$

$\Rightarrow f(n1) = 6m-5$.

If $n2=4m-1$, then $f(n2) = C(n2) / 2 \wedge 1$

$\Rightarrow f(n2) = 3(n2)+1/2 \wedge 1$

$\Rightarrow f(n2) = 6m-1$.

If $n3^*=8m-3$, then

$f(n3^*) = 3(n3^*) / 2 \wedge 2i$

Or

$f(n3^*) = 3(n3^*)+1/ 2 \wedge 2i -1$.

Therefore, the final table of proof as shown below.

n1	n2	n3*	C (n)	f (n1)	f (n2)	f(n3 *)
1			4	1		
	3		10		5	
		5	16			1
	7		22		11	
9			28	7		
	11		34		17	
		13	40			5
	15		46		23	
17			52	13		
	19		58		29	
		21	64			1
:	:	:	:	:	:	:..etc.

■Lemma theorem :- 1

Exactly one set (j) of odd, positive integers maps to 1, regardless if counter examples exist or not.

proof:-

¥ i € W / and let set (j) = n, then
If n=1, then C (1)=3 (1)+1/2^1 =1
: n=5, then C (5) = 3 (5)+1/2^4 =1
: n=21, then C (21)=3 (21)+1/2^6=1
: n=85, then C (85) =3 (85)+1/2^8 =1
: n=341, then C (341)=3 (341)+1/2^10 =1
: n = 1365,then C (1365)=3(1365)+1/2^ 12=1
: : : : : : :
: : : : : : :...etc.

Therefore, ¥ i € W/ and
if n= £2^2i, then
C (n) = 3 (£ 2 ^ 2i) + 1 / 2 ^ 2i + 2 = 1 always. ( )

■ Lemma theorem:-2

Let y be a range element of the 3n+1 function. Then for each finite exponent sequence i €/ N, there exist an X that maps to y via(i) possiblely followed by a" buffer " exponent.

Proof:-

¥ i, m €/ N and Let the fixed set Y €{C(x1), C(x2), C(n3), C(n4), C(n5), C(n6)},then there exist a fixed set X €{x1, x2 ,x3, x4, x5, x6} respectively, where

x1= 12m-9 € X maps C (x1)=18m- 13 € Y.

x2=12m -5 € X maps C(x2) = 18m- 7 € Y.

x3 = 12m-1 € X maps C (x3) = 18m-1 € Y.

x4= 24m-15 € X maps C (x4) = 18m-11 € Y.

x5=24m-7 € X maps C (x5) = 18m - 5 € Y.

x6=24m+ 1 € X maps C (x6)= 18m + 1 € Y.

● Note:-

I) If X €{ x1, x3, x5}, then

C (X)=3(X)+1/2^2i-1. (Alway have Odd exponent .)

II) If X €{ X2, x4, x6}, then

C (X)=3 (X)+1/2^ 2i. (Always have even exponent.)

Now, let me construct a proof table 1. 3 -(a) and (b)for the Lemma theorem :-2 by six fixed range element of set Y .

[redacted]able 1.3(a):- ¥i € Odd,/N & i € / N, then
C (X) = 3 (X) + 1 / 2 ^ 2i -1, Where
x1 = 12m -9,
x3=12m-5,
x5=12m -1,and
X € { x1, x3, x5} respectively.

x1 -> C (x1)	x3-> C (x3)	x5 ->C (x5)
3•••••••••••>5*	7••••••••••>11	11••••••••••>17
15•••••••••> 23	19••••••••> 29*	23••••••••••> 35
27•••••••••> 41	31••••••••> 47	35•••••••••>53*
39••••••••••> 59	43•••••••••>65	47••••••••••>71
51••••••••••>77*	55•••••••••> 83	59••••••••••> 89
63•••••••••> 95	67••••••> 101*	71••••••••••>107
75•••••••••>113	79•••••••>119	83•••••••••> 125*
87•••••••••>131	91•••••••>137	95••••••••> 143
99••••••••>149*	103•••••>155	107••••••> 161
111••••••>167	115••••••>173*	119••••••••> 179
123••••••>185	127••••••>191	131••••••••>197*
135•••••••> 203	139••••••> 209	143•••••••> 215
147•••••••> 221*	151•••••••> 227	155••••••••> 233
159•••••••> 239	163•••••••> 245*	167••••••••> 251
171••••••••> 257	175••••••••> 263	179••••••••> 269*
: :	: :	: :
: :	: :	: :....etc.

■able 1.3 (b) :- ¥ i € even, / N &m € / N, then
C (X) = 3 (X) + 1 / 2 ^ 2i,where

x2=24m -15,
x4=24m -7,
x6= 24m + 1, and
X € { X2, x4, x6 } respectively.

x2. C (x1)	x4. C (x4)	x6. C (x6)
9••••••••> 7	17••••••••> 13*	25•••••••••> 19
33••••••••> 25	41••••••••> 31	49••••••••••> 37*
57•••••••> 43	65••••••••> 49	73••••••••••> 55
81•••••••> 61*	89••••••••> 67	97•••••••••••> 73
105•••••> 79	113••••••> 85*	121••••••••> 91
129••••••>97	137•••••••> 103	145•••••••> 109*
153•••••> 115	161••••••••> 121	169•••••••> 127
177•••••> 133*	185••••••>139	193•••••••> 145
201••••••> 151	209••••••> 157	217•••••••> 163
225••••••> 169	233••••••> 175	241••••••> 181*
249•••••••> 187	257•••••••> 193	265•••••> 199
273••••••> 205*	281••••••> 211	289••••••> 217
297•••••••> 223	305•••••••> 229*	313•••••••> 235
321••••••> 241	329•••••••> 247	337•••••••> 253*
: :	: :	: :
: :	: :	: :...etc.

■Propositions:-
From table 1.3- (a)and (b), I can find out two proposition that helps to indicate what ever you insert any Odd natural numbers in the collatz, (3n + 1) conjecture,we obtain a unique mapping wrt any other Odd natural numbers. Thus, let me state the two proposition.

◆Proposition:- I
¥i,m,x1,x2,x3 €/N,& X€{x1,x3,x5};and x1 ≠x3 ≠x5,then
C(x1)/2^2i-1≠C(x3)/2^2i-1≠C(x5)/2^2i-1,
where x1=12m-9,x2=12m-5,x3=12m-1respectively.

Proof:-
Suppose x1≠x3 ≠x5,then
C(x1)/2^2i-1≠C(x3)/2^2i-1≠C(x3)/2^2i-1
=>3x1+1/2^2i-1≠3x2+1/2^2i-1≠3x3+1/2^2i-1
=>x1 ≠x3 ≠x5. ....(True!)
Conversly,assume that x1=x3=x5,then
C(x1)/2^2i-1=C(x2)/2^2i-1=C(x3)/2^2i-1.
=>3x1+1/2^2i-1=3x3+1/2^2i-1=3x5+1/2^2i-1
=>x1=x3=x5. ......(Contradiction!).Therefore,
¥i,m,x1,x3,x5€/N,& X€{x1,x3,x5},and x1≠x3≠x5,then
C(x1)/2^2i-1≠C(x3)/2^2i-1≠C(x5)/2^2i-1.[■]

◆Proposition :- II
¥ i, m, x2, x4, x6 € / N, &X € { X2, x4, x6 };and x2≠x4≠x6, thenC (x2) / 2 ^ 2i ≠C (x4) / 2 ^ 2i≠C (x6)/2 ^ 2i.

Proof:-
Follw the same procedure for this proof.

Generalization of the Collatz, (3n + 1) conjecture

The other all Odd, positive integers that are not listed in table 1.3 - (a) and (b) can be obtained from the six fixed set of elements in the domain X by multiple 4 and add 1.
For instance, X € { x1, x3, x5 }, then

a] x1'=4 (x1)+1=48m -35 € X

b] x3'=4 (x3)+1=48m - 19 € X

C] x5'=4 (x5)+1=48m - 3 € X.
This implies that X € { x1',x3', x5'},and maps with the fixed set of range of Y, whereY € { C (x1), C (x3), C (x5)}, because (x1& x1'), (x3 & x3'), (x5& x5') maps with the same range of Odd, positive integers Y,but with different exponents in the collatz, (3n + 1) conjecture.

Example:-1
¥ m € / N, & let i be exponent(exp), then
○m=1, x1=3 € X maps C (x1) / 2 ^ 2 i-1=5 € Y &i=1.
○m =1, x1'=4 (x1)+1=13 € X maps to
C (x1) / 2 ^ 2 i-1=5 € Y & i=3.
○m=13, x1''=4 (x1')+1=53 € X maps to
C (x1)/2 ^ 2i-1=5 € Y& i=5.
○m=53, x1'''=4 (x1'')+1=213 € X maps to
C (x) /2 ^ 2i - 1=5 € Y and i=7..........etc.

Note:-
The main thing I construct the following some table's 1. 3 - (a), (b), (c), (d), (e),& (f) is that to show the reformulated collatz , (3n + 1) conjecture holds an infinitely many odd natural numbers that having with an infinitely many power in base 2 that maps with another Odd natural numbers and they can be easily reach to 1.
Because, ¥ m, n € / N, the Collatz, (3n + 1) conjecture is an elements of the function f1 = 3n -2.

Continue from table 1.3 - (a) and (b). .....

Table 1.3 - (c)

¥ m €/ N , i=(Odd exp)=3, and
let $x1'=4 (x1) + 1=48m-35$
$x3'=4 (x3)+ 1=48m - 19$
$x5'= 4 (x5)+1=48m - 3$, and $X \in \{ x1', x3', x5' \}$.

x1'-> C (x1)	x3'-> C (x3)	x5'-> C (x5)
13---> 5*	29--> 11	45---> 17
61---> 23	77--> 29*	93---> 35
109--> 41	125-->47	141--->53*
157-->59	173--> 65	189---> 71
205--> 77*	221--> 83	237---> 89
253--> 95	269--> 101*	285--->107
301--> 113	317--> 119	333---> 125*
349--> 131	365--> 137	381---> 143
397--> 149*	413--> 155	429---> 161
445--> 167	461--> 173*	477---> 179
493--> 185	509--> 191	525---> 197*
541--> 203	557--> 209	573---> 215
589--> 221*	605-->227	621---> 233
637--> 239	653--> 245*	669--->251
685--> 257	701--> 263	717---> 269*
: :	: :	: :
: :	: :	: :...etc.

[redacted]able 1.3 - (d)

¥m €/ N and i=(even exp) = 4, then
x2'=4 (x2)+1=96m-59,
x4'=4 (x4)+1 = 96m-27,
x6'=4 (x6)+1=96m+5, and X €{ x2',x4', x6'} that maps to
the fixed set of range element of Y only one time.

x2' -> C (x2')	x4'-> C (x4')	x6'-> C (x6')
37------> 7	69••••••>13*	101---->19
133----> 25	165••••••>31	197---->37*
229----> 43	261•••••••>49	293---->55
325---> 61 *	357•••••••>67	389---->73
421----> 79	453•••••>85*	485--->91
517----> 97	549•••••>103	581--->109*
613----> 115	645•••••>121	677--->127
709---> 133 *	741••••>139	773--->145
805--->151	837••••> 157*	869---> 163
901--->169	933••••> 175	965---->181*
997--->187	1029•••> 193	1061--->199
1093--->205*	1125••••> 211	1157--->217
1189--->223	1221••••> 229*	1253---->235
1285--->241	1317••••> 247	1349---->253*
1381---> 259	1413-->265	1445--->271
1477--->277*	1509-->283	1541--->289
1573---> 295	1605-->301*	1637--->307
1605---> 313	1701-->329	1733--->325*
: :	: :	: :
: :	: :	: :.etc.

[redacted]able 1.3 - (e)

¥m €/ N, i =(Odd exp)=5, and
let x1'''=4 (x1'') + 1 = 192 m-139,
x3'''=4 (x3'') + 1=192m-75,
x5'''=4 (x5'')+ 1 = 192m-11, and X €{ x1''', x3''', x5''}.
Then all Odd, positive integers in the domain of X maps to the fixed set of range element in Y only one times.

x1'''-> C (x1)	x3'''-> C (x3)	x5'''-> C (x5)
53•••••> 5*	117••••> 11	181•••>17
245••••>23	309••••>29*	373•••••>35
437••••>41	501••••>47	565•••••>53*
629••••>59	693••••>65	757•••••>71
821••••>77*	885••••>83	949••••••>89
1013•••>95	1077••>101*	1141••••>107
1205•••>113	1269•••>119	1333••••>125*
1397•••>131	1461•••>137	1525••••>143
1589•••>149*	1653•••>155	1717••••>161
1781•••> 167	1845•••>173*	1909•••>179
1973•••>185	2037•••>191	2101•••>197*
2165•••>203	2229•••>209	2293•••>215
2357•••>221*	2421•••>227	2485•••>233
2549•••>239	2613•••>245*	2677•••>251
2741•••>257	2805•••>263	2869•••>269*
2933•••>275	2997•••>281	3061••>287
3125•••>293*	3189•••>299	3253••>305
3317•••>311	3381•••>317*	3445••>323
3509•••>329	3573•••>335	3637••>341*
: :	: :	: :
: :	: :	: :....etc.

[redacted]able 1.3 - (f)

¥m €/ N ,i =(even exp) = 6, and
Let x2'''=384m-235,
x4'''=384m-107,
x6'''=384m+21, and X €{ X2''', x4''',x6'''}, then

x2'''••> C(x2''')	x4'''••> C (x4''')	x6'''••>C (x6''')
149•••> 7	277•••> 13*	405•••>19
533••> 25	661•••> 31	789••>37*
917••> 43	1045•••>49	1173••>55
1301••>61*	1429•••> 67	1557••>73
1685••> 97	1813•••> 85*	1941••>91
2069••> 115	2197•••> 103	2325••>109*
2453••> 133*	2581•••> 121	2709••>127
2837••> 151	2965•••> 139	3093••>145
3221••> 169	3349•••>157*	3477••>163
3605••> 187	3733•••>175	3861••>181*
3989••> 205*	4117•••>193	4245••>199
4373••>223	4501•••>211	4629••>217
4757••> 241	4885•••>229*	5013••>235
5141••>259	5269•••>247	5397••>253*
5525••>277*	5653•••>265	5781••>271
5909••> 295	6037•••>28	6165••>289
6293••> 313	6421•••>301*	6549••>307
6677••> 313	6805••> 319	6933••> 325*
: :	: :	: :
: :	: :	: : ..etc.

★Appendix

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★ The End★