A new approach to prime numbers and why Golbach’s conjecture is true

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Abstract: A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors. Primes can thus be considered the “basic building blocks”, the atoms, of the natural numbers.

There are infinitely many primes, as demonstrated by Euclid around 300 BC. There is no known simple formula that separates prime numbers from composite numbers. However, the distribution of primes, that is to say, the statistical behavior of primes in the large, can be modelled. The first result in that direction is the prime number theorem, proven at the end of the 19th century, which says that the probability that a given, randomly chosen number $n$ is prime is inversely proportional to its number of digits, or to the logarithm of $n$.

The way to build the sequence of prime numbers uses sieves, an algorithm yielding all primes up to a given limit, using only trial division method which consists of dividing $n$ by each integer $m$ that is greater than 1 and less than or equal to the square root of $n$. If the result of any of these divisions is an integer, then $n$ is not a prime, otherwise it is a prime.

This paper introduces a new way to approach prime numbers, called the DNA-prime structure because of its intertwined nature, and a new process to create the sequence of primes without direct division or multiplication, which will allow us to introduce a new primality test, and a new factorization algorithm.

As a consequence of the DNA-prime structure, we will be able to provide a potential proof of Golbach’s conjecture.

A. INTRODUCTION

1. Prime sequence

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself. The sequence of prime numbers is infinite, is composed only by odd numbers and there is no known formula to generate it.

The first 25 prime numbers are given by:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, …

The infinite sequence has been studied in many different ways:

2. Differences between consecutive primes

The differences between two of these consecutive primes is calculated to be:

\[
\begin{array}{cccccccccccccccc}
1 & 2 & 2 & 4 & 2 & 6 & 4 & 6 & 2 & 6 & 4 & 6 & 2 & 6 & 4 & 6 & 4 & 6 & 8 \\
\end{array}
\]

With:

\[ g_n = p_n - p_{n-1} \]

Verifying that:

\[ \lim_{n \to \infty} g_n = \infty \]

And:

\[ \lim_{n \to \infty} \frac{g_n}{p_n} = 0 \]

The differences between primes are increasing and the prime number theorem proved that these gaps grows with the logarithm of \( n \). The function is neither multiplicative nor additive.

As of March 2017 the largest known prime gap with identified probable prime gap ends has length 5103138, with 216849-digit probable primes found by Robert W. Smith.[3] This gap has merit M=10.2203. The largest known prime gap with identified proven primes as gap ends has length 1113106, with 18662-digit primes found by P. Cami, M. Jansen and J. K. Andersen [4][5]

3. Ratios between consecutive primes

The ratios between two consecutive primes is given by:

\[
\begin{array}{cccccccccccccccc}
2 & 3 & 5 & 7 & 11 & 13 & 19 & 23 & 29 & 31 & 37 & 41 & 47 & 53 & 59 \\
1.50 & 1.67 & 1.40 & 1.57 & 1.18 & 1.46 & 1.21 & 1.26 & 1.07 & 1.19 & 1.11 & 1.15 & 1.13 & 1.11 \\
\end{array}
\]

These ratios are decreasing with:

\[ \lim_{n \to \infty} \frac{p_n}{p_{n-1}} = 1 \]
The gaps are not consistently decreasing and important research has been done on the limits of those gaps. This research is related to the counting of the number of primes less than a given number.

4. **Known algebraic expressions for primes**

The following expression by Euler connects all prime numbers with the Riemann Zeta function over all natural numbers:

\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - \frac{1}{p^s}} \]

One of the most important advance in the study of Prime numbers was the paper by Bernhard Riemann in November 1859 called “Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse” (On the number of primes less than a given quantity) [2].

In this paper, Riemann gave a formula for the number of primes less than \( x \) in terms the integral of \( 1/\log(x) \) and the roots (zeros) of the zeta function in the complex plane, defined by:

\[ \zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} \]

In this paper, Riemann formulated his famous hypothesis that all zeros of \( \zeta(z) \) have \( \text{Re}(z)=1/2 \).

In a paper not published yet [12] “An Engineer’s approach to the Riemann Hypothesis and why it is true” (Feb 2017) I formulated that the following expressions that are valid for all the non-trivial zeros of Riemann \( \zeta(z) \):

1. Any zero of \( \zeta(z) \) with \( z=\alpha+i\beta \) meet these two conditions:
   - (condition 1) \( \alpha=1/2 \)
   - (condition 2) Calculating \( \beta \):
     
     If \( S = \frac{1}{[\beta^2 + 1/4]} \) then for \( n=1/S \) →

     \[
     \left( \sum_{k=1}^{n} \sum_{j \neq k} k^{-1/2} * j^{-1/2} * \cos \left( \beta \left( \ln \left( \frac{k}{j} \right) \right) \right) \right) = 0
     \]

     This is a finite sum, with \( n \in \left[ 1, \frac{1}{S} \right] \)

2. All zeros of \( \zeta(z) \) are related through the following algebraic expression where \( z_1=\alpha_1+i\beta_1 \) and \( z_2=\alpha_2+i\beta_2 \) are non-trivial zeros of Riemann \( \zeta(z) \):

\[
\frac{n}{[\beta_2^2+(1-\alpha)^2]} \sum_{k=1}^{n} \sum_{j \neq k} k^{-1/2} * j^{-1/2} * \cos(\beta_2 \left( \ln \left( \frac{k}{j} \right) \right)) =
\]
\[
\frac{n}{[\beta^2+(1-\alpha)^2]} - \sum_{k=1}^{n} \sum_{j \neq k} k^{-1/2} * j^{-1/2} * \cos(\beta_1 (\ln \left(\frac{k}{j}\right)))
\]

when \( n \to \infty \).

3. The harmonic function \( H_n \) can be expressed in infinite ways as a function of each non-trivial zeros of Riemann \( \zeta(z) \):

\[
H_n = \frac{n}{[\beta^2+(1-\alpha)^2]} - \sum_{k=1}^{n} \sum_{j \neq k} k^{-1/2} * j^{-1/2} * \cos(\beta \ln \left(\frac{k}{j}\right))
\]

when \( n \to \infty \).

4. The Imaginary part of the non-trivial zeros of the Riemann \( \zeta(z) \) is equal to the zeros of the following function in the Real plane:

\[
P(n,\beta) = \sum_{k=1}^{n} \sum_{j \neq k} k^{-1/2} * j^{-1/2} * \cos \left( \beta \ln \left(\frac{k}{j}\right) \right)
\]

where \( 1/2+\beta i \) would be a non-trivial zeros of Riemann \( \zeta(z) \).

In the same paper, the author also provided a potential proof of the Riemann Hypothesis.

5. **Number of primes less than a given number**

Let’s call \( \pi(n) \) the number of primes less than \( n \). The prime number theorem says that:

\[
\lim_{n \to \infty} \frac{\pi(n)}{\ln n} = 1
\]

Which can be written as:

\[
\lim_{n \to \infty} \frac{\pi(n)}{\text{li}(n)} = 1
\]

Where:

\[
\text{li}(n) = \int_0^n \frac{dt}{\ln t}
\]

The following table shows the results of this approximation [6 modified]:
The effort in this direction is to find more accurate approximations to $\pi(n)$. All these expressions involve complex algebraic expressions of $\ln(n)$, or the Riemann Zeta function, and $\text{li}(x)$.

As an example, the Riemann hypothesis is equivalent to a much tighter bound on the error in the estimate for $\pi(n)$, and hence to a more regular distribution of prime numbers. Specifically, \[ |\pi(n) - \text{li}(x)| < \frac{1}{8\pi} \sqrt{x} \ln x \quad \text{for all } x > 2657. \]

### 6. Prime factorization

In number theory, the prime factors of a positive integer are the prime numbers that divide that integer with no remainder.

If $n$ is divided by $p$, there is a $k, r \in \mathbb{Z}$ such that:

$$n = k \cdot p + r$$

$p$ is a prime factor of $n$, if and only if $r = 0$, which can also be expressed using the mod(ulo) function by:

$$n \mod p = 0$$

Where the function $\mod(ulo)$ is defined as follows:

$$r = p - n \cdot \text{trunc}(\frac{p}{n})$$

The prime factorization of a positive integer is a list of the integer's prime factors, together with their multiplicities; the process of determining these factors is called integer factorization. The fundamental theorem of arithmetic says that every positive integer has a single unique prime factorization.[7]
To find prime factors requires to know if a number is a prime. This type of test is called primality test. Among other fields of mathematics, it is used extensively in cryptography.

The RSA codes in cryptography consists of very large composite numbers that have exactly two prime factors. These numbers are called semiprimes. Finding those two factors require very complex algorithms as the numbers are composed by two prime numbers of more than one hundred digits. As an example:

\[
\text{RSA-220} = \\
260138526203405784941654048610197513508038915719776718321197768109445641 \\
817966676608593121306582577250631562886676970448070001811149711863002112 \\
487928199487482066070131066586646083327982803560379205391980139946496955261
\]

\[
\text{FACTOR 1 of RSA-220} = \\
686365641226756627438237149928843780013084223997916484462124499332154106 \\
14414642667938213644208420192054999687
\]

\[
\text{FACTOR 2 of RSA-220} = \\
329290743948634981204930154921293529191645519653623395246268605116929034 \\
9309465246337824866390738191765712603
\]

The simple factorization method is the trial division method which consists in dividing sequentially by all known primes until we find a factor. Then we reduce the number by the factor and start again. This method is unpractical for large primes.

The fastest-known fully proven deterministic algorithm is the Pollard-Strassen method (Pomerance 1982; Hardy et al. 1990). [8]

Wolfram Math World mentions the following list of factorization methods: [9]

- Brent's Factorization Method,
- Class Group Factorization Method,
- Continued Fraction Factorization Algorithm,
- Direct Search Factorization,
- Dixon's Factorization Method,
- Elliptic Curve Factorization Method,
- Euler's Factorization Method,
- Excludent Factorization Method,
- Fermat's Factorization Method,
- Legendre's Factorization Method,
- Number Field Sieve,
- Pollard p-1 Factorization Method,
- Pollard rho Factorization Algorithm,
B. A new approach to Prime Numbers. Defining the DNA-Prime Sequences P+ and P-

So far, the sequence of primes is a mystery. We have been able to find some patterns, predict the number of primes, factor very large integers but we are still in the same position as Eratosthenes was more than 2000 years ago, when he introduced his sieve to identify prime numbers.

In this paper, we want to introduce a new way to look at primes and develop a new primality test and factorization method from it.

One characteristic that all primes greater than 3 have is that they can be expressed with one of the two following expressions:

\[ p = 6k_n + 1 \quad k_n \in N \]  \hspace{1cm} (1)

\[ p = 6k_m - 1 \quad k_m \in N \]  \hspace{1cm} (2)

In theory, if we knew the sequences \( k_n \) and \( k_m \) we would know all primes.

In this paper, we are going to provide a unique formulation for the two “generator” series \( k_n \) and \( k_m \).

As we can see in the following table:
The prime numbers belong to either $P^+$ or $P^-$ series.

It is amazing that all primes can be generated from the numbers 1, 2, 3, as:

\[
\begin{align*}
P^+ & \quad p = 1 + 2 \times 3 \times k_n \\
P^- & \quad p = -1 + 2 \times 3 \times k_m
\end{align*}
\]

I will call these two sequences the DNA-Prime Sequences as it resembles the intertwined DNA helix.

### C. Characteristics of the DNA-Prime Sequences

a. The difference between two primes in either sequence $P^+$ and $P^-$ is a multiple of 6. In the following table we show the two DNA-prime series, the difference between two consecutive elements of the series, the difference divided by 6 and the cumulative difference divided by 6.

We must observe that the cumulative difference from any element of the series and the first element, that we will call $R_n$ for the $P^+$ series, and $R_m$ for the $P^-$ series, is equal to the K generator minus 1. This key fact will help us formulate a way to generate the prime sequence.
Table 3

We can see in the chart that:

<table>
<thead>
<tr>
<th>P+</th>
<th>P-(n)-P-(n-1)</th>
<th>(P(n)-P(n-1))/6</th>
<th>Rn</th>
<th>P-</th>
<th>P(m)-P(m-1)</th>
<th>(P(m)-P(m-1))/6</th>
<th>Rm</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>11</td>
<td>2</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>17</td>
<td>3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>31</td>
<td>5</td>
<td>12</td>
<td>2</td>
<td>23</td>
<td>4</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>37</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>29</td>
<td>5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>41</td>
<td>7</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>61</td>
<td>10</td>
<td>18</td>
<td>3</td>
<td>47</td>
<td>8</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>67</td>
<td>11</td>
<td>6</td>
<td>1</td>
<td>53</td>
<td>9</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>12</td>
<td>6</td>
<td>1</td>
<td>59</td>
<td>10</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>79</td>
<td>13</td>
<td>6</td>
<td>1</td>
<td>71</td>
<td>12</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>97</td>
<td>16</td>
<td>18</td>
<td>3</td>
<td>83</td>
<td>14</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>103</td>
<td>17</td>
<td>6</td>
<td>1</td>
<td>89</td>
<td>15</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>109</td>
<td>18</td>
<td>6</td>
<td>1</td>
<td>101</td>
<td>17</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>127</td>
<td>21</td>
<td>18</td>
<td>3</td>
<td>107</td>
<td>18</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>139</td>
<td>23</td>
<td>12</td>
<td>2</td>
<td>113</td>
<td>19</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>151</td>
<td>25</td>
<td>12</td>
<td>2</td>
<td>131</td>
<td>22</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>157</td>
<td>26</td>
<td>6</td>
<td>1</td>
<td>137</td>
<td>23</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>163</td>
<td>27</td>
<td>6</td>
<td>1</td>
<td>149</td>
<td>25</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>181</td>
<td>30</td>
<td>18</td>
<td>3</td>
<td>167</td>
<td>28</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>193</td>
<td>32</td>
<td>12</td>
<td>2</td>
<td>173</td>
<td>29</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3

We can see in the chart that:

- P+ series: \( \text{P(n)-P(n-1)} \) mod 6 = 0
- P- series: \( \text{P(m)-P(m-1)} \) mod 6 = 0

b. The difference between any two primes in either series is given by:

\[
P^+ \text{ series: if } p_1=6^*k_1+1 \quad \text{and} \quad p_2=6^*k_2+1
\]
\[
P^- \text{ series: if } p_1=6^*k_1-1 \quad \text{and} \quad p_2=6^*k_2-1
\]

Then \( p_2-p_1 = 6^* (k_2-k_1) \)

c. The difference between any prime in either sequence P+ and P and the first one in the series is a multiple of the generators:

- P+ series: \( P_n = 7 + 6^*R_n \)
- P- series: \( P_m = 5 + 6^*R_m \)

Where \( R_n = k_n-1 \) and \( R_m = k_m-1 \)
If we can find a formula for the generators, then we will have a formula for the primes. Let’s take a look at the $R_n$ and $R_m$ sequences eliminating (in color) all those that don’t generate a prime using previous formulas:

D. The prime series can be formulated algebraically

From the tables above and testing many potential combinations, we have concluded that the sequence of DNA-primes and their generators $R_n$ and $R_m$ can be formulated algebraically as follows:

<table>
<thead>
<tr>
<th>$P^*$ series</th>
<th>$P_n = 7 + 6*R_n$</th>
<th>$P^*$ series</th>
<th>$R_n \neq x + (6x + 7) * y \quad x &gt; 0, y &gt; 1 \in N$</th>
<th>$R_n \neq -x + (6x - 7) * y \quad x &gt; 1, y &gt; 1 \in N$</th>
<th>$P^*$ series</th>
<th>$P_m = 5 + 6*R_m$</th>
<th>$P^*$ series</th>
<th>$R_m \neq x + (6x + 5) * y \quad x &gt; 0, y &gt; 1 \in N$</th>
<th>$R_m \neq -(x + 1) + (6x + 1) * y \quad x &gt; 0, y &gt; 1 \in N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^*$ series</td>
<td>$P_n = 1 + 6*K_n$ with $K_n=R_n+1$</td>
<td>$P^*$ series</td>
<td>$P_n = 1 + 6*K_n$ with $K_n=R_n+1$</td>
<td>$P^*$ series</td>
<td>$P_n = 1 + 6*K_n$ with $K_n=R_n+1$</td>
<td>$P^*$ series</td>
<td>$P_n = 1 + 6*K_n$ with $K_n=R_n+1$</td>
<td>$P^*$ series</td>
<td>$P_n = 1 + 6*K_n$ with $K_n=R_n+1$</td>
</tr>
</tbody>
</table>

Table 4

Table 5
The first numbers in the generator series are $R_n$ and $R_m$:

$R_n = 1, 2, 4, 5, 6, 9, 10, 11, 12, 15, 16, 17, 20, 22, 24, 25, 26, 29, 31, 32, 34, 36, 37, 39, 44, 45, 46, 50, 51, 54, 55, 57, 60, 61, 62, 65, ...$

$R_m = 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 16, 17, 18, 21, 22, 24, 27, 28, 29, 31, 32, 37, 38, 39, 41, 42, 43, 44, 46, 48, 51, 52, 57, 58, 59, 63, ...$

And the primes generated by these generators:

$\text{Primes generated} = [2, 3, 5, 7], 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, ...$

We prepared a simple code to run the prime generation and we this is a test for the prime numbers less than $n=1,000,000$

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Version 03/14/2017

-------------------------------
GENERATING PRIME SERIES Start @ 2017-03-21 11:44:11
-------------------------------
Number of Primes = 78498
Number of Prime Pairs = 8168 $\%\Pi(x) = 10.405360646131111$
-------------------------------
My series generated @ 2017-03-21 11:49:27
--- GENERATED Prime list is correct
-------------------------------
Last prime position: 78498 --> 999983
-------------------------------
GENERATING PRIME SERIES Ends @ 2017-03-21 11:49:27
-------------------------------

E. The gaps between elements of the DNA-prime series

The gaps between elements of $Rn$ and $Rm$ can be described using the Prime Number Theorem as the DNA-prime series and the prime series have a bijective relationship.

We can affirm that:
\[ R_n - R_{n-1} \sim \frac{1}{6} \ln(6R_n + 7) < R_{n-1} < R_n \]

\[ R_m - R_{m-1} \sim \frac{1}{6} \ln(6R_m + 5) < R_{m-1} < R_m \]

F. **DNA-Prime twins**

The definition of the two DNA-prime series implies that any \( S_{2n} \) such that \( S_{2n} = R_n = R_m \) will create a pair of twin primes.

The list of the first \( S_{2n} \) is:

\[ S_{2n} = [1, 2, 4, 6, 9, 11, 16, 17, 22, 24, 29, 31, 32, 37, 39, 44, 46, ...] \]

And the pairs generated:

- Twin Primes from \( S_{2n} \) = 1 = [11, 13]
- Twin Primes from \( S_{2n} \) = 2 = [17, 19]
- Twin Primes from \( S_{2n} \) = 4 = [29, 31]
- Twin Primes from \( S_{2n} \) = 6 = [41, 43]
- Twin Primes from \( S_{2n} \) = 9 = [59, 61]
- Twin Primes from \( S_{2n} \) = 11 = [71, 73]
- Twin Primes from \( S_{2n} \) = 16 = [101, 103]
- Twin Primes from \( S_{2n} \) = 17 = [107, 109]
- Twin Primes from \( S_{2n} \) = 22 = [137, 139]
- Twin Primes from \( S_{2n} \) = 24 = [149, 151]
- Twin Primes from \( S_{2n} \) = 29 = [179, 181]
- Twin Primes from \( S_{2n} \) = 31 = [191, 193]
- Twin Primes from \( S_{2n} \) = 32 = [197, 199]
- Twin Primes from \( S_{2n} \) = 37 = [227, 229]
- Twin Primes from \( S_{2n} \) = 39 = [239, 241]
- Twin Primes from \( S_{2n} \) = 44 = [269, 271]
- Twin Primes from \( S_{2n} \) = 46 = [281, 283]

We must add [3,5] and [5,7] to complete the series.

The series \( S_{2n} \) has infinite terms as \( R_n \) and \( R_m \) are also infinite and the condition for \( R_n = R_m \) does not have a maximum limit. The generation of \( R_n \) and \( R_m \) is done over all \( x, y \in N \). The proof will only require to define a \( x, y \) that makes all 4 conditions of the DNA-series not equal. The length of the gap among twin prime sets can be calculated from the difference between elements in \( S_{2n} \).
The Brun’s theorem [11] proved that the sum of the reciprocals of these twin primes converges to a finite value known as Brun’s constant, usually denoted by $B_2$. In 2002 Pascal Sebah and Patrick Demichel used all twin primes up to $10^{16}$ to give the estimate:

$$B_2 = \sum_{p, p+2 \text{ pairs}} \left( \frac{1}{p} + \frac{1}{p+2} \right) = 1.902160583104 \quad [6]$$

We can express this equation with the DNA-prime expression as:

$$B_2 = \sum_{0, R = R_n = R_m} \left( \frac{1}{6R+7} + \frac{1}{6R+5} \right) = 1.902160583104 \quad [6]$$

Adding $\left( \frac{1}{3} + \frac{1}{5} \right)$.

Using again the definition of the two DNA-prime series we can calculate other partial consecutive sequences of primes $S_{kn}$. For instances, series of 4 consecutive primes will require that $S_{4n}$ and $S_{4n+1}$ to be both in $R_n$, and $R_m$.

The list of the first $S_{4n}$ is:

$S_{4n} = [1, 16, 31, 136, 246, 311, 346, 541, 576, 941, 1571, \ldots]$  

With:

$S_{4n} = 1 = [11, 13, 17, 19]$  
$S_{4n} = 16 = [101, 103, 107, 109]$  
$S_{4n} = 31 = [191, 193, 197, 199]$  
$S_{4n} = 136 = [821, 823, 827, 829]$  
$S_{4n} = 246 = [1481, 1483, 1487, 1489]$  
$S_{4n} = 311 = [1871, 1873, 1877, 1879]$  
$S_{4n} = 346 = [2081, 2083, 2087, 2089]$  
$S_{4n} = 541 = [3251, 3253, 3257, 3259]$  
$S_{4n} = 576 = [3461, 3463, 3467, 3469]$  
$S_{4n} = 941 = [5651, 5653, 5657, 5659]$  
$S_{4n} = 1571 = [9431, 9433, 9437, 9439]$  

We must add $[5, 7, 11, 13]$ to complete the series.

There is also a Brun’s constant for these prime quadruplets with $B_4 = 0.87058 838\ldots$ Wolf derived an estimate for the Brun-type sums $B_n$ of $4/n$.

I have not found any elements for sets for $S_{6n}$ and above yet.
G. DNA-Prime primality test

For a number N to be prime, the following conditions must be met:

a) If (N-1) mod 6≠0 and (N+1) mod 6≠0 the number is not prime

b) If (N-1) mod 6=0 then N=6*k_n+1 and R_n=K_n-1

   CONDITION 1 \((R_n - 7s) mod (6s + 1) \neq 0\) for \(s \in N<k_n\)

   CONDITION 2 \((R_n + 7s) mod (6s - 1) \neq 0\) for \(s \in N<k_n\)

   If s=1 or s=k_n then N is Prime.

c) If (N+1) mod 6=0 then N=6*k_m+1 and R_m=K_m-1

   CONDITION 3 \((R_m - (s - 1)) mod (6s - 1) \neq 0\) for \(s \in N<k_m\)

   CONDITION 4 \((R_m - 5s) mod (6s + 1) \neq 0\) for \(s \in N<k_m\)

   If s=1 or s=k_m then N is Prime.

Examples:

<table>
<thead>
<tr>
<th>Kn</th>
<th>Km</th>
<th>Rn</th>
<th>Rm</th>
<th>s</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,489</td>
<td>748.00</td>
<td>747.00</td>
<td>11</td>
<td>1</td>
<td>No prime</td>
</tr>
<tr>
<td>6,839</td>
<td>1139.67</td>
<td>1140.00</td>
<td>163</td>
<td>3</td>
<td>No prime</td>
</tr>
<tr>
<td>9,973</td>
<td>1662.00</td>
<td>1662.33</td>
<td>1662</td>
<td></td>
<td>Prime</td>
</tr>
</tbody>
</table>

The maximum values of \((s)\) for each condition can be formulated as a function of \(N\) to optimize the algorithm.

H. DNA-Prime factorization process

For any given number N, the factors can be calculated with the following method:

1. Check if the number is divisible by 2,3,5,7
   a. If yes, divide N by factors and continue with \(N^*=N/factors\) \((2^a,3^b,5^c,7^d)\)

2. If \(N^*>7\), Determine if \(N^*\) belongs to any DNA-Prime Sequence \(P^+\) or \(P^-\)
   a. If \(N \in P^+\) then calculate \(k_n=(N-1)/6\) and \(R_n=K_n-1\) and check Condition 1
      i. If Condition 1 is not met at \(s<k_n\) then \((6*s-1)\) is a prime factor of \(N^*\). Divide \(N^*\) by the factor. \(N^{**}=N^*/factor\)
      ii. If condition 1 is met the check Condition 2
1. If Condition 2 is not met at \( s < k_n \) then \((6s+1)\) is a prime factor of \( N^* \). Divide \( N^* \) by the factor. \( N^{**} = N^*/\text{factor} \)

2. If condition 2 is met, \( N^* \) is prime
   
   b. If \( N \in P^- \) then calculate \( k_m = \frac{(N+1)}{6} \) and \( R_m = k_m - 1 \) and check Condition 3
      
      i. If Condition 3 is not met at any \( s < k_m \) then \((6s-1)\) is a prime factor of \( N^* \). Divide \( N^* \) by the factor. \( N^{**} = N^*/\text{factor} \)
      
      ii. If condition 3 is met then Check Condition 4

   1. If Condition 4 is not met at \( s < k_m \) then \((6s+1)\) is a prime factor of \( N^* \). Divide \( N^* \) by the factor. \( N^{**} = N^*/\text{factor} \)

   2. If condition 4 is met, \( N^* \) is prime
   
   c. If \( N^* \) is not prime run process for new \( N^{**} = N^*/\text{factor} \)

Some examples of factorization with computer time:

1. **Factoring \( N = 1234567890 \) showing all the steps:**

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   FACTORIZA Version 03/14/2017
   ---------------------------------------------
   FACTORIZA Start @  2017-03-20 19:03:22

   \( N = 1234567890 \)
   --- PARTIAL N= 6172839450 \( \rightarrow \) FACTORS+ [2]
   --- PARTIAL N= 2057613150205761315 \( \rightarrow \) FACTORS+ [2, 3]
   --- PARTIAL N= 41152230041152263 \( \rightarrow \) FACTORS+ [2, 3, 5]
   --- N= 41152230041152263 is NOT P+ or P-
   --- PARTIAL N= 137174210013717421 \( \rightarrow \) FACTORS+ [2, 3, 5, 3]
   --- N= 137174210013717421 is in Series: P+

   --- PARTIAL N= 38738833666681 \( \rightarrow \) FACTORS+ [2, 3, 5, 3, 3541]
   --- N= 38738833666681 is in Series: P+

   --- N= 38738833666681 \( \rightarrow \) Failed Condition 1+ \( @ s = 590 \)
   --- PARTIAL N= 38738833666681 \( \rightarrow \) FACTORS+ [2, 3, 5, 3, 3541]
   --- N= 38738833666681 is in Series: P+

   --- N= 38738833666681 \( \rightarrow \) Failed Condition 1+ \( @ s = 601 \)
PARTIAL N= 10739903983 \rightarrow \text{FACTORS}^+ [2, 3, 5, 3, 3541, 3607]

PARTIAL N= 10739903983 is in Series: P^+

N= 384103 \rightarrow \text{FACTORS}^+ [2, 3, 5, 3, 3541, 3607, 27961]

PARTIAL N= 384103 is in Series: P^+

PARTIAL N= 3803 \rightarrow \text{FACTORS}^+ [2, 3, 5, 3, 3541, 3607, 27961, 101]

PARTIAL N= 3803 is in Series: P^-

PARTIAL N= 3803 \rightarrow \text{FACTORS}^+ [2, 3, 5, 3, 3541, 3607, 27961, 101]

N= 12345678901234567891 \rightarrow \text{FACTORS} = 2^5 \cdot 10133 \cdot 503779 \cdot 75576521

N= 12345678901234567892 \rightarrow \text{FACTORS} = 2^2 \cdot 3086419725308641973

N= 12345678901234567893 \rightarrow \text{FACTORS} = 3 \cdot 14210467 \cdot 289591207693

N= 12345678901234567894 \rightarrow \text{FACTORS} = 2 \cdot 17 \cdot 31 \cdot 43 \cdot 1189 \cdot 229099455043

N= 12345678901234567895 \rightarrow \text{FACTORS} = 5 \cdot 9577219 \cdot 257813440441

N= 12345678901234567896 \rightarrow \text{FACTORS} = 2^3 \cdot 3^2 \cdot 5 \cdot 19 \cdot 389 \cdot 757 \cdot 7177 \cdot 1830053

N= 12345678901234567897 \rightarrow \text{FACTORS} = 2^3 \cdot 3 \cdot 5 \cdot 101 \cdot 3541 \cdot 3607 \cdot 3803 \cdot 27961

N= 12345678901234567898 \rightarrow \text{FACTORS} = 2 \cdot 6172839450617283949

N= 12345678901234567899 \rightarrow \text{FACTORS} = 3^2 \cdot 392 \cdot 11261 \cdot 73039 \cdot 168787409

N= 12345678901234567900 \rightarrow \text{FACTORS} = 2^2 \cdot 5^2 \cdot 11 \cdot 12517 \cdot 22147 \cdot 40486211

N= 12345678901234567901 \rightarrow \text{FACTORS} = 3^4 \cdot 227 \cdot 1676593797071

N= 12345678901234567902 \rightarrow \text{FACTORS} = 2^2 \cdot 3 \cdot 13^2 \cdot 12175225740862493

N= 12345678901234567903 \rightarrow \text{FACTORS} = 2 \cdot 59 \cdot 2917 \cdot 10247749396943

N= 12345678901234567904 \rightarrow \text{FACTORS} = 2^5 \cdot 10133 \cdot 503779 \cdot 75576521

N= 12345678901234567905 \rightarrow \text{FACTORS} = 3 \cdot 5 \cdot 4314587 \cdot 190758758621

N= 12345678901234567906 \rightarrow \text{FACTORS} = 2^2 \cdot 3^2 \cdot 715629579 \cdot 520498309

N= 12345678901234567907 \rightarrow \text{FACTORS} = 1151 \cdot 1223 \cdot 8770274702459

N= 12345678901234567908 \rightarrow \text{FACTORS} = 2^2 \cdot 3^4 \cdot 22727 \cdot 1676593797071

N= 12345678901234567909 \rightarrow \text{FACTORS} = 488899381 \cdot 25251983089

2. Factorization of 20 consecutive numbers without factorization steps:

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FACTORIZA Version 03/14/2017

FACTORIZA Start @ 2017-03-20 22:37:51

N= 12345678901234567890 \rightarrow \text{FACTORS} = 2 \cdot 3^2 \cdot 5 \cdot 101 \cdot 3541 \cdot 3607 \cdot 3803 \cdot 27961

N= 12345678901234567891 \rightarrow \text{PRIME

N= 12345678901234567892 \rightarrow \text{FACTORS} = 2^2 \cdot 3086419725308641973

N= 12345678901234567893 \rightarrow \text{FACTORS} = 3 \cdot 14210467 \cdot 289591207693

N= 12345678901234567894 \rightarrow \text{FACTORS} = 2 \cdot 17 \cdot 31 \cdot 43 \cdot 1189 \cdot 229099455043

N= 12345678901234567895 \rightarrow \text{FACTORS} = 5 \cdot 9577219 \cdot 257813440441

N= 12345678901234567896 \rightarrow \text{FACTORS} = 2^3 \cdot 3 \cdot 5 \cdot 19 \cdot 389 \cdot 757 \cdot 7177 \cdot 1830053

N= 12345678901234567897 \rightarrow \text{FACTORS} = 6373 \cdot 1937184826805989

N= 12345678901234567898 \rightarrow \text{FACTORS} = 2 \cdot 6172839450617283949

N= 12345678901234567899 \rightarrow \text{FACTORS} = 3^2 \cdot 11261 \cdot 73039 \cdot 168787409

N= 12345678901234567900 \rightarrow \text{FACTORS} = 2^2 \cdot 5^2 \cdot 11 \cdot 12517 \cdot 22147 \cdot 40486211

N= 12345678901234567901 \rightarrow \text{FACTORS} = 314312807 \cdot 39278319643

N= 12345678901234567902 \rightarrow \text{FACTORS} = 2 \cdot 3 \cdot 13^2 \cdot 12175225740862493

N= 12345678901234567903 \rightarrow \text{FACTORS} = 2 \cdot 59 \cdot 2917 \cdot 10247749396943

N= 12345678901234567904 \rightarrow \text{FACTORS} = 2^5 \cdot 10133 \cdot 503779 \cdot 75576521

N= 12345678901234567905 \rightarrow \text{FACTORS} = 3 \cdot 5 \cdot 4314587 \cdot 190758758621

N= 12345678901234567906 \rightarrow \text{FACTORS} = 2 \cdot 23 \cdot 315629579 \cdot 520498309

N= 12345678901234567907 \rightarrow \text{FACTORS} = 1151 \cdot 1223 \cdot 8770274702459

N= 12345678901234567908 \rightarrow \text{FACTORS} = 2^2 \cdot 3^4 \cdot 22727 \cdot 1676593797071

N= 12345678901234567909 \rightarrow \text{FACTORS} = 488899381 \cdot 25251983089
I. **Use of DNA-Prime to prove Golbach’s conjecture**

The Golbach’s conjecture says that every even integer greater than 2 can be expressed as the sum of two primes.[10]

From the definition of the two DNA-Prime sequences we know that any prime can be expressed as:

\[ p = 6k_n + 1 \quad k_n \in \mathbb{N} \]
\[ p = 6k_m - 1 \quad k_m \in \mathbb{N} \]

The addition of two odd prime numbers will always be even.

If \( N=2q \) is any even number, for it to be the addition of two primes, the following needs to be true:

\[ N = 2q = p_1 + p_2 \]

The possible combinations of odd numbers that added together amount to \( N \) are:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a+b=N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N-1</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>N-3</td>
<td>N</td>
</tr>
<tr>
<td>5</td>
<td>N-5</td>
<td>N</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N-1</td>
<td>1</td>
<td>N</td>
</tr>
</tbody>
</table>

To illustrate the problem, let’s build a simple table for \( N=18 \)

<table>
<thead>
<tr>
<th>N</th>
<th>p1</th>
<th>p2</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>18</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>18</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>18</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>18</td>
<td>15</td>
<td>3</td>
</tr>
</tbody>
</table>
We can see that there are \( \frac{N}{2} \) combinations of two odd numbers that add up to \( N \).
We can also see that there are 2 combinations involving 1 and 1 is not a prime.
The option \( \frac{N}{2} + \frac{N}{2} = N \) does not involve addition of primes and we can disregard it.

Of the remaining combinations, they repeat themselves due to the commutative property of the addition in \( N \).

So, the net number of potential valid combinations of two odd numbers with one of them at least being prime is:

\[
\frac{N}{2} - \frac{3}{2}
\]

If \( p \) is prime, based on the Prime number theorem, we can see that for \( N > 76 \) the number of combinations is larger than the number of primes \( < N \) as:

\[
\frac{N}{2} - \frac{3}{2} > \frac{N}{\ln N} \quad \text{for } N > 76
\]

So, the number of primes that meet Golbach’s conjecture for any even number \( N \) are proportionally less than the number of combinations of odd numbers as \( N \) grows.

We know that if \( p_1 \) and \( p_2 \) are primes, we can use the DNA-Prime series to say:

\[
p_1 = 6 \times k_1 \pm 1 \]
\[
p_2 = 6 \times k_2 \pm 1
\]

And there are three possibilities:

\[
N = p_1 + p_2 = 6 \times (k_1 + k_2) - 2
\]
\[
N = p_1 + p_2 = 6 \times (k_1 + k_2)
\]
\[
N = p_1 + p_2 = 6 \times (k_1 + k_2) + 2
\]

Based on this, and assuming that \( p_1 \) and \( p_2 \) exist, we can affirm that:
If \( N \mod 6 = 0 \)  
\( N \) is the addition of a \( p_1 \in P^+ \) and \( p_2 \in P^- \)

If \( (N-2) \mod 6 = 0 \)  
\( N \) is the addition of a \( p_1 \in P^+ \) and \( p_2 \in P^+ \)

If \( (N+2) \mod 6 = 0 \)  
\( N \) is the addition of a \( p_1 \in P^- \) and \( p_2 \in P^- \)

given that for any even number, there is a \( q \in N \) such that \( N = 2q \) and the previous expressions are equivalent to:

\[
q \mod 3 = 0 \\
\text{or } \ (q-1) \mod 3 = 0 \\
\text{or } \ (q+1) \mod 3 = 0
\]

Which is obviously true as for any 3 consecutive numbers \( q-1, q, (q+1) \), one of them must necessarily be divisible by 3.

The three possible combinations of primes mentioned earlier can be also reformulated as follows:

If \( q \mod 3 = 0 \)  
\[
\frac{q}{3} = k_1 + k_2 \\
p_1 = 6k_1 + 1 \text{ and } p_2 = 6k_2 - 1
\]

If \( (q-1) \mod 3 = 0 \)  
\[
\frac{q-1}{3} = k_1 + k_2 \\
p_1 = 6k_1 + 1 \text{ and } p_2 = 6k_2 + 1
\]

If \( (q+1) \mod 3 = 0 \)  
\[
\frac{q+1}{3} = k_1 + k_2 \\
p_1 = 6k_1 - 1 \text{ and } p_2 = 6k_2 - 1
\]

As examples of these expressions:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( q )</th>
<th>( q \mod 3 )</th>
<th>Potential Primes</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>27</td>
<td>9</td>
<td>2 ( P^+ ) 7 ( P^- )</td>
<td>13</td>
<td>41</td>
</tr>
<tr>
<td>64</td>
<td>32</td>
<td>11-1</td>
<td>3 ( P^- ) 8 ( P^- )</td>
<td>17</td>
<td>47</td>
</tr>
<tr>
<td>68</td>
<td>34</td>
<td>11+1</td>
<td>5 ( P^+ ) 6 ( P^+ )</td>
<td>31</td>
<td>37</td>
</tr>
</tbody>
</table>
To prove Golbach’s conjecture we must prove that for any \( n \in N \) we can find combinations of \( R_n \) and \( R_m \) in the DNA-Prime generator series such that:

\[
q = k_n + k_m = R_n + R_m - 2
\]

or

\[
q = k_{n1} + k_{n2} = R^1_n + R^2_n - 2
\]

or

\[
q = k_{m1} + k_{m2} = R^1_m + R^2_m - 2
\]

In other words, that for any \( q \in N \), we can find two elements of \( R_n \), or two elements of \( R_m \), or one element of \( R_n \) and one element of \( R_m \), that add up to \( q \), as all even numbers are of the form: \( N=2q \), with \( q \in N \)

To prove it, we are going to use an induction proof.

We will define a condition that is observable and met for a certain \( k=k^* \), we will assume that the condition is met at \( k=n-1 \) and then we will prove that this means the condition is also true at \( k=n \) for any element \( n \) of the generator series.

Let’s observe that in the following chart of \((R_m+R_m)\), the square \( R_m*R_m \) contains at least all naturals up to \( R_m \).

<table>
<thead>
<tr>
<th>Rm x Rm table</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>\hline 1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>
For example, the square for \( (R_{11} \times R_{11}) = (11 \times 11) \) contains up to the natural number up to 11, i.e. contains \([1], 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\)

We could use any other cardinal to observe that this true.

Let’s assume now that the condition is true for \( R_{m-1} \), which means that the square \( R_{m-1} \times R_{m-1} \) contains all naturals up to \( R_{m-1} \) generated by the addition of two given \( R^1_m \) and \( R^2_m \) both \( < R_{m-1} \) and let’s prove that the condition is met for the square \( R_m \times R_m \), which means that the square \( R_m \times R_m \) must contain all naturals up to \( R_m \).

The set of natural numbers between \( R_{m-1} \) and \( R_m \) are, by definition of the matrix \( R_m \times R_m \):

\[
D_n = \{ R_m - R_{m-1} \} = \{ R_{m-1} + 1, R_{m-1} + 2, R_{m-1} + 3 \ldots, R_{m-1} + (R_m - R_{m-1}) \}
\]

We know that all naturals up to \( R_{m-1} \) exists and for each \( n < R_{m-1} \) there are two:

\[
R^j_{m-1} = \{ R^1_{m-1} \ldots R^j_{m-1} \}
\]

\[
R^k_{m-1} = \{ R^1_{m-1} \ldots R^k_{m-1} \}
\]

Such that \( n = R^j_{m-1} + R^k_{m-1} \)

The assumption over \( R_{m-1} \) implies that the set of :\( \{ R^j_{m-1} + R^k_{m-1} \} \) generates all naturals up to \( R_{m-1} \)

\[
\{ R^j_{m-1} + R^k_{m-1} \} = \{ 2, 3, 4, 5, \ldots, R_{m-1} \}
\]

with \( R^j_{m-1} = \{ 1, 2, 3, 4, 5, 6, 8, 9, \ldots, R_{m-1} \} \)

If we add \( R_{m-1} \) to any of the two \( R^j_{m-1} + R^k_{m-1} \) we can say that:

\[
\{ R_{m-1} + R^j_{m-1} \} = \{ R_{m-1} + 1, R_{m-1} + 2 \ldots, 2 \times R_{m-1} \}
\]

\( 2 \times R_{m-1} \) is the last diagonal term of the defined and known matrix \( R_{m-1} \times R_{m-1} \) which is contained in \( R_m \times R_m \).

And we know from [D] that \( R_{m-1} < 2 \times R_{m-1} \) so, therefore, \( D_n \) is contained in the matrix \( R_m \times R_m \).
Same proof works for \((R_n+R_n)\) and \((R_n+R_m)\), which proves the Golbach’s conjecture.

As an example, to find the primes that add up to 180:

\[
N=180 \quad N \mod 6 = 0 \quad \text{so one prime belongs to } P^+ \text{ and the other to } P^- \\
q=N/6=30 \quad \text{and } R_n+R_m=30-2=28
\]

All the combinations of \(R_n+R_m = 28\) are:

\[
\begin{align*}
\text{Rn} & \quad \text{Rm} & \quad P^+ & \quad P^- \\
1 & \quad 27 & \quad 13 & \quad 167 = 180 \\
4 & \quad 24 & \quad 31 & \quad 149 = 180 \\
6 & \quad 22 & \quad 43 & \quad 137 = 180 \\
10 & \quad 18 & \quad 67 & \quad 113 = 180 \\
11 & \quad 17 & \quad 73 & \quad 107 = 180 \\
12 & \quad 16 & \quad 79 & \quad 101 = 180 \\
15 & \quad 13 & \quad 97 & \quad 83 = 180 \\
17 & \quad 11 & \quad 109 & \quad 71 = 180 \\
20 & \quad 8 & \quad 127 & \quad 53 = 180 \\
22 & \quad 6 & \quad 139 & \quad 41 = 180 \\
24 & \quad 4 & \quad 151 & \quad 29 = 180 \\
25 & \quad 3 & \quad 157 & \quad 23 = 180 \\
26 & \quad 2 & \quad 163 & \quad 17 = 180
\end{align*}
\]

Table 7

J. **Open challenges**

- Can a formula for \(\pi(x)\) counting all primes up to \(x\) be formulated exactly based on the 4 conditions of the DNA-prime series? To calculate this, we should be able to calculate the amount of repetitions among the four conditions.
- Can a formula for gaps between primes and twin primes be formulated based on the 4 conditions of the DNA-prime series?

Thanks

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REFERENCES

(1) Computer Programs used in this proof designed and developed by Pedro J. Caceres


(3) http://trnicely.net/index.html#TPG


(5) Jump up^ A proven prime gap of 1113106

(6) "A table of values of π(x)". Xavier Gourdon, Pascal Sebah, Patrick Demichel. Retrieved 2008-09-14.


(9) http://mathworld.wolfram.com/PrimeFactorizationAlgorithms.html


(12) Pedro Caceres, “An Engineer’s approach to the Riemann Hypothesis and why it is true”, February 2017. HTTP://viXra.org/abs/1703.0104